BOOLEAN RINGS AND COHOMOLOGY

A. D. WALLACE

This note is a comment on a paper of Franklin Haimo [1]. A Stone space is a compact totally disconnected Hausdorff space. Let $B(S)$ be the Boolean ring of the Stone space S. The multiplication in $B(S)$ is intersection and the addition is symmetric difference. Thus $B(S)$ is isomorphic with the ring of all maps (=continuous functions) $\phi: S \rightarrow I_2$, I_2 being the integers mod 2 with the discrete topology. Using the Alexander-Kolmogoroff groups (Spanier [2]) it is readily seen that $\phi \in Z^0(S)$, the group of 0-cocycles of S with I_2 as coefficient group, if and only if ϕ is a map. Using ordinary multiplication of functions in $Z^0(S)$ it is at once clear that $B(S) \approx H^0(S)$, since $Z^0(S) = H^0(S)$. If $\{S_\lambda, \pi_{\lambda\mu}\}$ is an inverse system of Stone spaces, then inv lim S_λ is a Stone space. Using Steenrod's continuity theorem (see [2]) we have

$$^0(\text{inv lim } S_\lambda) \approx \text{dir lim } H^0(S_\lambda)$$

and hence

$$B(\text{inv lim } S_\lambda) \approx \text{dir lim } B(S_\lambda).$$

It might be remarked that, since a Stone space S is compact and totally disconnected, $H^0(S) = H(S)$, the cohomology ring of S, because $H^n(S) = 0$ for $n > 0$.

Bibliography

1. Franklin Haimo, *Some limits of Boolean algebras*, Proceedings of the American Mathematical Society vol. 2 (1951) pp. 566–576.

2. E. H. Spanier, *Cohomology theory for general spaces*, Ann. of Math. vol. 49 (1948) pp. 407–427.

The Tulane University of Louisiana

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