THEOREM 10. *For the derivatives of functions $f_p(z)$ of class S :*

$$1 - |z| \leq |f'_p(z)| \leq 1 + |z|.$$

These inequalities are sharp. They are attained by $f_p(z) = z - z^2/2$ at $z = \pm r$, r real.

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A REMARK ON REVERSIBLE MATRICES

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The matrix $A = (a_{nk})$ is called *reversible* if for each $y = \{y_n\} \in (c)$ the equations $y_n = \sum_{k=0}^{\infty} a_{nk}x_k$ ($n=0, 1, \dots$) have exactly one solution $x = \{x_k\}$. In this case there exist [1; 4] constants c_k, b_{kn} with $\sum_n |b_{kn}| < \infty$ for each k , such that

$$(1) \quad x_k = c_k \lim_{n \rightarrow \infty} y_n + \sum_{n=0}^{\infty} b_{kn} y_n \quad (k = 0, 1, \dots).$$

It is further stated in [1, p. 50] that the c_k are bounded. This is questioned in [4], where it is pointed out that if the c_k were generally bounded they would have to be almost all zero, but this remark does not dispose of the matter, for it might conceivably be a true theorem that for each reversible matrix the c_k are almost all zero. (For row-finite matrices, all c_k vanish; [3].) The example given in [4, p. 47] seems inconclusive. The purpose of this note is to show by a very simple example that in fact the c_k need not be bounded.

Consider the transformation

$$y_{2m} = \sum_{p=0}^m x_{2p}, \quad y_{2m+1} = 2^{-m} x_{2m+1} + \sum_{p=0}^{\infty} x_{2p},$$

where $m=0, 1, \dots$. For each $y \in (c)$ we have

$$x_{2m+1} = 2^m (y_{2m+1} - \lim_n y_n);$$

thus $c_{2m+1} = -2^m$ is not bounded.

This has a bearing on a paper [2] in which the following theorem is stated:

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THEOREM. *A reversible conservative matrix A sums a divergent sequence if and only if $\|A^{-1}\| = \infty$.*

Here $\|A^{-1}\|$ is defined as $\sup_k (|c_k| + \sum_n |b_{kn}|)$ with c_k, b_{kn} as above, and the theorem is proved under the assumption that the c_k are bounded. There is, however, no difficulty in allowing for the possibility of the c_k being unbounded. For then in the *sufficiency* part of the proof [2, p. 915], we note that if $\|B\| = \sup_k \sum_n |b_{kn}| < \infty$, we may take $y = \{1, 1, 1, \dots\}$, and obtain at once from (1) a divergent sequence x which is A -summable, while if $\|B\| = \infty$, we can find by classical methods a sequence $y = \{y_n\}$ with $\lim_n y_n = 0$ and By unbounded. For such a sequence the c_k are immaterial and we have simply $x = By$. This completes the proof.

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