$$(7) \quad P = \iiint k\rho(\nabla u)^2 dx dy dz$$

are simultaneously extremized with respect to variations in u . From a physical viewpoint, this amounts to extremizing the energy stored, and the power dissipated in the region under consideration.

The Euler-Lagrange equations for this problem are equations (1) and (4) which, as shown above, are equivalent to equation (5). Thus, a third-order differential equation, namely equation (5), is a necessary condition for an extremum in this problem.

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ON BERNSTEIN'S SELF-DUAL SET OF POSTULATES FOR BOOLEAN ALGEBRAS

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In 1916 Professor B. A. Bernstein published an elegant, self-dual set of postulates for Boolean algebras.¹ It was asserted at that time that the postulates were independent, but Mr. Robert Russell of the University of California discovered that the independence proofs for the commutative laws were erroneous.² It has remained an open question whether each of these laws actually is independent of the other laws. The answer, as this note will establish, is negative.

We quote Bernstein's postulates, omitting those concerned with closure under the fundamental operations:

P1. $x + y = y + x$.	P3. $x + y \cdot z = (x + y) \cdot (x + z)$.
P2. $x \cdot y = y \cdot x$.	P4. $x \cdot (y + z) = x \cdot y + x \cdot z$.
P5. For every x , there is a c such that, for every u , $u + x \cdot c = u$ and $u \cdot (x + c) = u$.	
P6. There are x and y such that $x \neq y$.	

We shall derive P1 from P2-P5.

Given x , let c be an element such that, for every u , $u + x \cdot c = u$ and $u \cdot (x + c) = u$ (cf. P5).

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¹ B. A. Bernstein, *A simplification of the Whitehead-Huntington set of postulates for Boolean algebras*, Bull. Amer. Math. Soc. vol. 22 (1916) p. 458.

² This error was acknowledged by Professor Bernstein in his article, *A dual-symmetric definition of Boolean algebra free from postulated special elements*, Scripta Mathematica vol. 16 (1950) p. 157.

$$\begin{aligned} \text{T1. } x + (x + c) &= x + c. & x + (x + c) &= (x + (x + c)) \cdot (x + c) \\ &= [\text{P3}] x + (x + c) \cdot c = [\text{P2}] x + c \cdot (x + c) = x + c. \end{aligned}$$

$$\begin{aligned} \text{T2. } y \cdot x + y &= y. & y \cdot x + y &= y \cdot x + y \cdot (x + c) \\ &= [\text{P4}] y \cdot (x + (x + c)) = [\text{T1}] y \cdot (x + c) = y. \end{aligned}$$

$$\begin{aligned} \text{T3. } x \cdot (x \cdot c) &= x \cdot c. & x \cdot (x \cdot c) &= x \cdot (x \cdot c) + x \cdot c \\ &= [\text{P4}] x \cdot (x \cdot c + c) = [\text{P2}] x \cdot (c \cdot x + c) = [\text{T2}] x \cdot c. \end{aligned}$$

$$\begin{aligned} \text{T4. } y \cdot (y + x) &= y. & y \cdot (y + x) &= (y + x \cdot c) \cdot (y + x) \\ &= [\text{P3}] y + (x \cdot c) \cdot x = [\text{P2, T3}] y + x \cdot c = y. \end{aligned}$$

$$\begin{aligned} \text{T5. } x \cdot x &= x. & x \cdot x &= x \cdot x + x \cdot c \\ &= [\text{P4}] x \cdot (x + c) = x. \end{aligned}$$

$$\begin{aligned} \text{T6. } x + y &= (y + x) \cdot (x + y). & (y + x) \cdot (x + y) \\ &= [\text{P4}] (y + x) \cdot x + (y + x) \cdot y = [\text{P2}] x \cdot (y + x) + y \cdot (y + x) \\ &= [\text{T4}] x \cdot (y + x) + y = [\text{P4, T5}] (x \cdot y + x) + y = [\text{T2}] x + y. \end{aligned}$$

P1 now follows immediately from P2 and T6.

A dual derivation will clearly yield P2 from P1 and P3–P5. If, however, either P1 or P2 is eliminated, the resulting system will be independent, as Professor Bernstein's original remarks establish.

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