ON A CONVERGENCE PROBLEM

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Research Problem 25 [1] reads as follows:

If $a_1 < a_2 < \dots$ are positive integers, if C is compact, and if $\sin a_n x \rightarrow 0$ for all x in C , prove that the convergence must actually be uniform.

The following example shows that the "theorem" is false.

Let

$$a_n = 2^n \quad (n = 1, 2, \dots);$$

let C be the compact set of points

$$(0, \pi, \pi/2, \pi/4, \pi/8, \dots).$$

Obviously $\sin a_n x \rightarrow 0$ for each x in C , but for any $\epsilon < 1$ and any positive integer, N , there is an x in C (namely, $x = \pi/2^{N+2}$) such that

$$|\sin a_{N+1} x| = |\sin \pi/2| = 1 > \epsilon,$$

so that the convergence is not uniform.

REFERENCE

1. A. Shields, *Research Problem 25. A convergence problem*, Bull. Amer. Math. Soc. vol. 60 (1954) p. 589.

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