

PROCEEDINGS  
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EDITED BY

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WALTER RUDIN

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## Journals Published by the American Mathematical Society

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