(a) $((a+1)/z)a+(z-3)a$ when $a\equiv z-1 \pmod z$ and $a\geqq z^2-5z+3$,

(b) $[(a+1)/z](a+z)+(z-3)a$

when $a\not\equiv z-1 \pmod z$ and $a\geqq z^2-4z+2$.

We omit the proof of this result as it is rather long.

It is not hard to find the desired N for specific triples of numbers. For instance when a_0, $a_1=a_0+2$, $a_2=a_0+3$ we find the value of N to be $[x/3]\cdot x+2+x$. If the largest of a_0, a_1, a_2 is sufficiently larger than the other two and those two are relatively prime then the N is easily determined also. In fact if $a_0<a_1<a_2$ and $(a_0, a_1)=1$ and $a_2>(a_0-1)(a_1-1)-a_0$ then $N=(a_0-1)(a_1-1)$.

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ON THE INFINITUDE OF PRIMITIVE k-NONDEFICIENTS

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H. N. Shapiro has defined [1] a k-nondeficient as an integer n which satisfies

$$\sigma(n)/n \geqq k \qquad (k \text{ real}) \tag{1}$$

where $\sigma(n)$ is the sum of the divisors of n. Integers n which do not satisfy (1) are called k-deficient. A primitive k-nondeficient is defined as a k-nondeficient, all of whose proper divisors are k-deficient. In the same paper, Shapiro shows that, in order for an infinite number of primitive k-nondeficients to exist, it is necessary that k be of the form

$$\prod_{i=1}^{m} \frac{p_i^{\alpha_i+1}-1}{(p_i-1)p_i^{\alpha_i}} \prod_{i=m+1}^{n} \frac{p_i}{p_i-1} \tag{2}$$

or, written another way,

$$\prod_{i=1}^{m} \frac{\sigma(p_i^{\alpha_i})}{p_i^{\alpha_i}} \prod_{i=m+1}^{n} \frac{p_i}{p_i-1}$$

where $p_1, p_2, p_3, \cdots, p_n$ are distinct primes and $0\leqq m\leqq n$. In this note we show that, for every k of the form (2), an infinite number of

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primitive k-nondeficients of $n+1$ distinct prime factors do exist.

Given a k of the form (2) with $n \neq m$, consider any number

$$N_x = q_x \prod_{i=1}^{m} p_i^{\alpha_i} \prod_{i=m+1}^{n} p_i^{\beta_i},$$

where

(a) the p_i $(i=1, 2, 3, \cdots, n)$ and the α_i $(i=1, 2, 3, \cdots, m)$ are derived from k,

(b) the β_i are chosen sufficiently large so that

$$\frac{\sigma(N_x/q_x \cdot p_i)}{N_x/q_x \cdot p_i} \, (i = 1, 2, 3, \cdots, m)$$
$$< \frac{\sigma(N_x/q_x \cdot p_i)}{N_x/q_x \cdot p_i} \, (i = m+1, m+2, m+3, \cdots, n)$$

and

(c) q_x is a prime distinct from the p_i and chosen sufficiently large so that $\sigma(N_x)/N_x < k$. We shall show that N_x will become primitive k-nondeficient when multiplied by some factor of the form $\prod_{i=m+1}^{n} p_i^{\gamma_i}$, not all $\gamma_i = 0$. And, since the choice of numbers q_x is infinite, the process gives rise to the infinite set of primitive k-nondeficients we seek.

Consider the number $N_x(s)$ formed from N_x by increasing each β_i by some arbitrary number s. If s is sufficiently large, then

$$\frac{\sigma[N_x(s)]}{N_x(s)} \equiv \frac{q_x+1}{q_x} \prod_{i=1}^{m} \frac{p_i^{\alpha_i+1} - 1}{(p_i - 1)p_i^{\alpha_i}} \prod_{i=m+1}^{n} \frac{p_i^{\beta_i+s+1} - 1}{(p_i - 1)p_i^{\beta_i+s}}$$
$$> \prod_{i=1}^{m} \frac{p_i^{\alpha_i+1} - 1}{(p_i - 1)p_i^{\alpha_i}} \prod_{i=m+1}^{n} \frac{p_i}{p_i - 1} \equiv k,$$

and $N_x(s)$ is k-nondeficient.

It is readily shown that $N_x(s)$ has at least one primitive k-nondeficient divisor. And, by condition (b) on our choice of N_x and the fact that $\sigma(p^\delta)/p^\delta$ is less than $p/(p-1)$ for any p, δ, one such divisor is of necessity a multiple of $q_x \prod_{i=1}^{m} p_i^{\alpha_i}$. Thus it can be found by the process of first testing for deficiency the numbers $N_x(s)/p_{m+1}$, $N_x(s)/p_{m+1}^2$, $N_x(s)/p_{m+1}^3, \cdots, N_x(s)/p_{m+1}^j, \cdots, (j \leq \beta_{m+1}+s)$ until a deficient one is found, say $N_x(s)/p_{m+1}^{\eta_1}$. (η_1 will mean $\beta_{m+1}+s+1$ if no deficient number turns up.) The same test is then applied in turn to

$$N_x(s)/p_{m+1}^{\eta_1-1}$$

with powers of p_{m+2}, to

$$N_x(s)/(p_{m+1}^{\eta_1-1} \cdot p_{m+2}^{\eta_2-1})$$

with powers of $p_{m+3}, \cdots$, and finally to $N_x(s)/\prod_{i=1}^{n-m-1} p_{m+i}^{\eta_i-1}$ with powers of p_n. The number we seek is $N_x(s)/\prod_{i=1}^{n-m} p_{m+i}^{\eta_i-1}$.

For the case of a k of the form (2) with $n=m$, an infinite set of primitive k-nondeficients is easy to find—for example, the set of all numbers $N_x = q_x \cdot \prod_{i=1}^{n} p_i^{\alpha_i}$ where the q_x are sufficiently large to insure primitiveness and the p_i, α_i stem from k.

Reference

1. Harold N. Shapiro, *Note on a theorem of Dickson*, Bull. Amer. Math. Soc. vol. 55 (1949) pp. 450–452.

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A CLASS OF SIMPLE MOUFANG LOOPS

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1. Introduction. A Moufang loop is a loop that satisfies the associative identities

$$\text{(M)} \qquad xy\cdot zx = x(yz\cdot x);\ x(y\cdot xz) = (xy\cdot x)z;\ (zx\cdot y)x = z(x\cdot yx).$$

The only known examples of simple Moufang loops are the simple groups. In the present paper we will prove the following theorem.

Theorem. *Let R be a simple alternative, not-associative, ring possessing an idempotent not its unit element. Let L be the loop of all regular elements of R and let Z be the center of L. Then either L/Z is a simple, not-associative, Moufang loop or L/Z contains a simple, not-associative, Moufang subloop M which is a normal subloop of index* 2.

As we shall see in the course of our proof, the present theorem is a nonassociative analogue of the well known results on the special projective group $PSL(n, K)$ (see [**4**, p. 44]).

In §5, we shall prove that the Cayley-Dickson numbers of norm 1 over the real field R^* (modulo their center) are simple and indicate how this is the best possible result.

Our results will yield finite, not-associative, simple Moufang loops whose possible orders are $(2^{7n}-2^{3n})$ and $2^{-1}(p^{7n}-p^{3n})$ if p is an odd prime. Thus we obtain a simple, not-associative, Moufang loop of order 120.

Although we have tried to make this paper reasonably self contained, some of the results by Bruck (2) on Moufang loops will be used without reference.

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