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RAND CORPORATION

AN ADDENDUM TO "A NOTE ON SUBSEQUENCES"

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Some years ago, I obtained the following theorem [2]:

THEOREM. *A real sequence $\{x_n\}$ is convergent if there exists a regular matrix summability transform T which sums every subsequence of $\{x_n\}$.*

In proving this, the initial observation was made: "We may restrict ourselves to bounded sequences." Since then I have received a number of queries relating to this point, and have privately communicated the proof. Together with several other remarks, this is the substance of this note.

Let $\{x_n\}$ be unbounded, and let T be given by a matrix $[a_{nk}]$. We show that there is a subsequence $\{x_{r_n}\} = \{x'_n\}$ which is not summable T . In discussing the summability of unbounded sequences, the class of regular transforms falls roughly into two subclasses, those (like triangular methods) that apply to *all* unbounded sequences, and those that apply only to unbounded sequences that do not increase too rapidly. In the first case, we shall show that any unbounded sequence x has a subsequence x' whose transform $T(x')$ is unbounded (and certainly divergent); in the second case, we produce a subsequence x' which is not in the domain of T . (Wilansky has informed me that he also has a proof in the case in which T is triangular.)

CASE (i). Let $[a_{n,k}]$ have infinitely many rows which terminate in a string of zeros (have finite length). If T is regular, then the lengths of these cannot be uniformly bounded. Write $a(n, k)$ for $a_{n,k}$, and choose two strictly increasing sequences $\{n_j\}$, $\{k_j\}$ so that $a(n_j, k_j)$ is the last nonzero entry in the n_j th row. Since $x = \{x_n\}$ is unbounded,

Received by the editors January 12, 1956.

a subsequence $x' = \{x'_n\}$ can be selected, term by term, so that $|\sum_{k=1}^j a(n_j, k)x'_k| \geq j$, for $j = 1, 2, \dots$. If $y' = T(x')$, then $|y'_n| \geq j$ and y' is unbounded.

CASE (ii). If the first case does not apply, then $[a_{n,k}]$ has only a finite number of "short" rows—if $n \geq n_0$, $a(n, k) \neq 0$ for infinitely many k . Choose a subsequence x' of x , term by term, so that

$$\begin{aligned} |a(1, 1)|^2 |x'_1| &\geq |a(1, 1)|, \\ |a(1, 2)|^2 |x'_2| &\geq |a(1, 2)|, \quad |a(2, 2)|^2 |x'_2| \geq |a(2, 2)|, \end{aligned}$$

and in general,

$$|a(n, k)|^2 |x'_k| \geq |a(n, k)|, \quad n = 1, 2, \dots, k.$$

If $n \geq n_0$, we have $\limsup_{k \rightarrow \infty} |a(n, k)x'_k| \geq 1$, and $\sum_{k=1}^{\infty} a(n, k)x'_k$ diverges. The subsequence x' accordingly does not lie in the domain of T .

Comparing these, it is natural to ask if it is not possible to find, in case (ii), a subsequence x' in the domain of T , but whose transform $T(x')$ is divergent. This is not true in general; for example, if the original sequence grows too rapidly to belong to the domain of T , then the same will hold for every subsequence. However, the question may be asked with the modification that x be unbounded and itself summable T .

An extension of the theorem allowing bounded *complex* sequences was obtained by Agnew [1].

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