R. L. MOORE'S AXIOM 1′ AND METRIZATION

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Let S be a Hausdorff space for which there exists a simple sequence $G_1, G_2, \cdots$ of open coverings such that (1) for each n, $G_n \supset G_{n+1}$, and (2) if H and K are nonintersecting closed subsets of S one of which is compact, then for some n no element of G_n intersects both H and K.[2] At the 1957 Summer Meeting of the Society the question arose in connection with Mr. Armentrout's paper, *A study of certain plane-like spaces without the use of arcs*,[3] as to whether or not S when satisfying certain rather complicated axioms was metric. I remarked that there did exist such nonmetric spaces. This observation was incorrect.

THEOREM. *The space S is metric.*

PROOF. Let p be a point of an open set R. There exists a natural number n such that if $g, h \in G_n$, $p \in g$, and $g \cdot h \neq 0$, then $g+h \subset R$. For suppose, on the contrary, that for each natural number n, there exist $g_n, h_n \in G_n$, $p \in g_n$, $g_n \cdot h_n \neq 0$ and $(g_n+h_n)\cdot(S-R) \neq 0$; let p_n be a point of $g_n \cdot h_n$. Obviously $p_1, p_2, \cdots$ converges to p. Let $H = R\cdot(p+p_1+p_2+\cdots)$ and let $K = S-R$. Both H and K are closed and H is compact. Furthermore, for each n some element of G_n intersects both H and K. This is a contradiction.

It now follows from Moore's metrization theorem [1] that S is metric.

REFERENCES

1. L. F. McAuley, *A relation between perfect separability, completeness, and normality in semi-metric spaces*, Pacific J. Math. vol. 6 (1956) pp. 315–326.

2. R. L. Moore, *Foundations of point-set theory*, Amer. Math. Soc. Colloquium Publications, vol. 13, New York, 1932.

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Presented to the Society, November 30, 1957; received by the editors, November 9, 1957.

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[2] Cf., Moore's Axiom 1′ in [2, p. 324].

[3] Abstract number 797, Bull. Amer. Math. Soc. vol. 63 (1957) p. 403