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ON A SPECIAL INTEGRAL EQUATION¹

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1. **Introduction.** R. H. Cameron posed the following question² in a paper [1]. Does

$$(1.1) \quad y(t) = x(t) + \int_0^t [x(s)]^2 ds, \quad 0 \leq t \leq 1,$$

have a solution $x \in C$ for almost every choice of $y \in C$? Here C denotes the space of continuous functions on $0 \leq t \leq 1$ which vanish at $t=0$, and "almost every" means all but a set of Wiener measure³ zero. The answer is no as we proceed to show.

2. We will show that if $y \in N = \{y \in C: |y(t) + 4t| < 1/10, 0 \leq t \leq 1\}$ then (1.1) has no solution x among the elements of C . Then the answer to the question is no, since N , a uniform neighborhood, has positive measure.

Suppose that $y \in N$, $x \in C$, and (1.1) holds. Let

$$Z(t) = \begin{cases} 0, & 0 \leq t \leq 1/10 \\ -4(t - 1/10), & 1/10 \leq t \leq \pi/4 + 1/10 \end{cases}$$

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² This question has two other formulations found in [1].

³ See, for instance [2].

$$W(t) = \begin{cases} 0, & 0 \leq t \leq 1/10 \\ -2 \tan 2(t - 1/10), & 1/10 \leq t < \pi/4 + 1/10 \end{cases}$$

$$E = \{t: x(t) \geq W(t), 1/10 \leq t < \pi/4 + 1/10\}$$

and $t_1 = \inf E$. It is easy to see that

$$\begin{aligned} 1/10 < t_1 < \pi/4 + 1/10, \\ [x(t)]^2 &\geq [W(t)]^2, \end{aligned} \quad 0 \leq t < t_1,$$

and

$$x(t_1) = W(t_1).$$

Since

$$Z(t) - \frac{3}{10} > y(t), \quad \frac{1}{10} \leq t < \frac{\pi}{4} + \frac{1}{10},$$

and

$$Z(t) = W(t) + \int_0^t [W(s)]^2 ds, \quad 0 \leq t < \pi/4 + 1/10$$

we have using (1.1)

$$x(t) = y(t) - \int_0^t [x(s)]^2 ds < Z(t) - \frac{3}{10} - \int_0^t [W(s)]^2 ds = W(t) - \frac{3}{10}$$

for $1/10 \leq t < t_1$. This is a contradiction since $x(t_1) = W(t_1)$.

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