A NOTE ON INTEGRAL TRANSFORMS

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To remove the restriction $f_n(1)=0$ in my paper [1], one sets

$$(1) \quad f_n(\sigma) = f_n(1)\sigma^{-n} + f_n^*(\sigma)$$

and

$$(2) \quad y_n(u) = z_n(u) + w_n(u),$$

and obtains from

$$(3) \quad \int_{\sigma}^1 \frac{T_n(u/\sigma) y_n(u) du}{(u^2 - \sigma^2)^{1/2}} = f_n(\sigma)$$

the equations:

$$(4) \quad \int_{\sigma}^1 \frac{T_n(u/\sigma) z_n(u) du}{(u^2 - \sigma^2)^{1/2}} = f_n(1)\sigma^{-n}$$

and

$$(5) \quad \int_{\sigma}^1 \frac{T_n(u/\sigma) w_n(u) du}{(u^2 - \sigma^2)^{1/2}} = f_n^*(\sigma).$$

The solution of (4) is found to be

$$(6) \quad z_n(u) = \frac{2f_n(1)}{\pi} \frac{T_{n-1}(u)}{(1 - u^2)^{1/2}}$$

by Lemmas 1, 2, and 3, while that of (5) is given in [1] as

$$(7) \quad w_n(u) = -\frac{2}{\pi} \int_u^1 \frac{T_{n-1}(u/v) d[v^n f_n^*(v)]}{v^{n-1}(v^2 - u^2)^{1/2}}.$$

Consequently, the solution of (3) is

$$(8) \quad y_n(u) = \frac{2f_n(1)}{\pi} \frac{T_{n-1}(u)}{(1 - u^2)^{1/2}} - \frac{2}{\pi} \int_u^1 \frac{T_{n-1}(u/v) d[v^n f_n^*(v)]}{v^{n-1}(v^2 - u^2)^{1/2}}.$$

REFERENCE

1. Ta Li, *A new class of integral transforms*, Proc. Amer. Math. Soc. vol. 11 (1960) pp. 290-298.

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Received by the editors September 6, 1960.