EACH HOMOGENEOUS NONDEGENERATE CHAINABLE CONTINUUM IS A COSET SPACE

TA-SUN WU¹

Let X be a compact Hausdorff homogeneous space and G be the full group of homeomorphisms from X to X . We denote the isotropy group at x by G_x . If G is topologized by uniform convergence [1], then it is well known that G is a topological group, and the function $\theta: G/G_x \rightarrow X$, defined by $\theta(gG_x) = g(x)$ is one-one and continuous. It is an open problem whether θ is a homeomorphism, i.e., whether X is a coset space.

In this note we prove that each homogeneous nondegenerate chainable continuum M is such a space.

Ford [2]² has proved that every SLH (strongly locally homogeneous) completely regular Hausdorff space is a coset space. We will prove that M is not an SLH space but still a coset space.

Bing [3] has proved that each homogeneous nondegenerate chainable space is a pseudo-arc. Therefore we only need to prove that the pseudo-arc is a coset space. In his papers [3; 4; 5], Bing gave a very beautiful treatment of the pseudo-arc. We will follow his definitions and notations, and we will not repeat all his definitions here.

Description of the pseudo-arc M [4, p. 730]: Let P and Q be two points of a compact metric space and $D_1, D_2, \dots$ a sequence of chains from P to Q such that for each positive integer i , (1) D_{i+1} is crooked in D_i , (2) no link of D_i has a diameter of more than $1/2^i$ and (3) the closure of each link of D_{i+1} is a subset of a link of D_i . Then M is the common part of $D_1^*, D_2^*, \dots$, where D_i^* denotes the sum of the links of D_i .

It is well known that the pseudo-arc is an indecomposable continuum [4].

DEFINITION. A space X is an SLH space if for any $x \in X$, and neighborhood V of x , there is a neighborhood U of x , $U \subset V$, such that for any $y \in V$, there is a homeomorphism g of X such that $g(x) = y$ and $g(z) = z$ if $z \notin U$.

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² The author takes the liberty here of mentioning that Theorem 4.1 in [2] is not correct. Counterexamples and some discussions are given in the author's paper mentioned in Footnote 1.

Since each composant of the indecomposable continuum X is dense in X and composants are preserved under homeomorphisms, the pseudo-arc M can not be an SLH space.

LEMMA. *If M is a pseudo-arc, then any open set U meets infinitely many (in fact uncountably many) different composants of M .*

PROOF. Since M has uncountably many composants, and each composant is dense in M .

PROPOSITION. *M is a coset space.*³

PROOF. Let G be the full group of homeomorphisms of M , which is topologized by uniform convergence. Let G_P be the isotropy group at P (cf. description of M). Since M is a compact metric space, we can define the (unique) uniformity generated by the family of sets $V_r = \{(x, y) | x, y \in M \text{ and } \rho(x, y) < r\}$ where $r = 1/2^n$, $n = 1, 2, \dots$, and $\rho(x, y)$ = distance between x and y . Let $S(r)$ be the set of all homeomorphisms g of M such that, for all $x \in M$, $\rho(x, g(x)) < r$. By the remark of the first section, all we need to show is that, if $r = 1/2^n$ for some n , there is an open set V containing P such that if $y \in V$, there is a homeomorphism $g \in S(r)$ such that $g(P) = y$.

Let $r = 1/2^n$. There is an i such that the diameter of each link of D_i is less than $r/4$. Let V be the first link of D_i , let y be any point of V , and select a point z of the last link of D_i such that z and y are in different composants (cf. Lemma).

Using the technique of the proof of Theorem 13 [4], we can find sequences $E_{1n}, E_{2n}, \dots$ ($n = 1, 2$) of chains closing down on M such that (1) E_{j1} is a chain from P to Q and E_{j2} is a chain from y to z , (2) $E_{11} = E_{12} = D_i$, (3) no element of E_{jn} has diameter greater than $1/2^{i+j}$, (4) $E_{j+1,n}$ is crooked in E_{jn} , (5) the closure of each element of $E_{j+1,n}$ is a subset of some element of E_{jn} , and (6) E_{jn} is a consolidation of some D_k .

Now using the techniques of the proof of Theorem 12 [4] followed by those of the proof of Theorem 11 [4] we arrive at a homeomorphism g of M such that, if x is a point of the j th element of D_i , $g(x)$ belongs to either the $(j-1)$ th, the j th or $(j+1)$ th element of D_i and $g(P) = y$. Clearly, $\rho(x, g(x)) < r$, this proves that $g \in S(r)$. Hence, by the remark above, we know that M is a coset space.

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³ The author is sincerely grateful for the referee's suggestions and comments on the proof of this proposition.

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TULANE UNIVERSITY AND
ST. MARY'S DOMINICAN COLLEGE

IMMERSIONS OF ALMOST PARALLELIZABLE MANIFOLDS

MORRIS W. HIRSCH¹

The purpose of this note is to prove the following theorem:

An almost parallelizable n -manifold M can be immersed in Euclidean q -space R^q if $2q > 3n$.

By *immersion* $f: M \rightarrow R^p$ we mean a continuously differentiable map whose Jacobian matrix has rank $n = \dim M$ at each point. We denote the *differential* of an immersion f by df .

A *regular homotopy* $f_t: M \rightarrow R^n$ is a homotopy such that each f_t is an immersion and df_t is a homotopy of the tangent bundle of M into R^n . In this case f_0 and f_1 have equivalent normal bundles.

We say M is *almost parallelizable* if the tangent bundle of $M - x$ is trivial, for some $x \in M$.

To prove the theorem, we first observe that if M is not compact, or is bounded, then M is parallelizable, and by [1, 6.3], M can be immersed in $R^{n+1} \subset R^q$. Hence we assume M is compact and unbounded. Let B be an n -ball diffeomorphically embedded in M , with bounding $(n-1)$ sphere S . Put $M_0 = M - \text{int } B$. By the remark above, there is an immersion $f: M_0 \rightarrow R^{n+1}$. We consider f as an immersion in R^q , and we deform f through a regular homotopy near S , keeping $f|S$ fixed, so that if X is a unit tangent vector to M at point $x \in S$ pointing into M_0 , then $df(X)$ is the unit vector $e = (0, \dots, 0, 1)$ normal to R^{q-1} in R^q . We still have $f(S) \subset R^{q-1}$.

Since the immersion f is regularly homotopic to an immersion $M \rightarrow R^{n+1}$, the normal bundle of f is trivial. This enables us to apply a lemma [2, 3.2] of M. Kervaire, which implies that the *Smale in-*

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