ON A THEOREM OF RUDIN AND KLEE

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Let X and Y be separable metric spaces, and $C(X, Y)$ the space of continuous functions from X to Y . In [2], M. E. Rudin and V. L. Klee proved that $C(X, Y)$ is hereditarily Lindelöf in the pointwise topology.² Using an idea obtained in conversation from Professor S. Mazur, we can actually show

THEOREM 1. *There exists on $C(X, Y)$ a topology $\mathfrak{J}$ with countable base which makes the map $(f, x) \rightarrow f(x)$ jointly continuous.³ Hence $\mathfrak{J}$ is finer than the compact-open topology, which is therefore—along with the pointwise topology—both hereditarily Lindelöf and hereditarily separable.*

PROOF. Let $\mathfrak{U}$ and $\mathfrak{V}$ be countable bases for X and Y , respectively, and let $\mathfrak{J}$ have as sub-base the collection of sets $\{f \in C(X, Y) \mid f(U) \subset V\}$, with $U \in \mathfrak{U}$ and $V \in \mathfrak{V}$. Then $\mathfrak{J}$ certainly has a countable base. Clearly $\mathfrak{J}$ also makes $(f, x) \rightarrow f(x)$ jointly continuous, and hence [1, p. 223] is finer than the compact-open topology. Finally, every subset of $C(X, Y)$ is certainly Lindelöf and separable for the countable-base topology $\mathfrak{J}$, and hence also for any coarser topology.

REFERENCES

1. J. L. Kelley, *General topology*, Toronto, Van Nostrand, 1955.
2. M. E. Rudin and V. L. Klee, *A note on certain function spaces*, Arch. Math. vol. 7 (1956) pp. 469–470.

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² The terminology is the same as in J. L. Kelley [1].

³ A metric giving such a topology was described to the author by Professor S. Mazur. The topology $\mathfrak{J}$ given in our proof is simpler, but in general not metrizable.