Baire spaces cannot be removed. If Q is the space of rationals in E_1 with the relative topology, there is a separately continuous $f: Q \times Q \rightarrow E_1$ which is zero on a dense subset of $Q \times Q$ but not identically zero.

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COMPLETE SEQUENCES OF FUNCTIONS¹

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Although the result of this note is implicitly contained in the work of A. A. Talalyan [2] and could also have been a corollary to the theorem in [1], it seems to be of sufficient interest to merit explicit treatment.

It is known (see [1]) that if $\{f_1, f_2, \dots, f_n, \dots\}$ is a sequence of measurable functions which is complete in the space M of measurable functions (i.e., every measurable f is the limit in measure of a sequence of finite linear combinations of $\{f_1, f_2, \dots, f_n, \dots\}$) then $\{f_2, f_3, \dots, f_n, \dots\}$ is also complete in M .

Let X be a separable Banach space of measurable functions on $[a, b]$ such that for every measurable $G \subset [a, b]$, with $m(G) > 0$, the set X_G of restrictions of the functions in X to G is a Banach space and

- (a) If $\{g_n\}$ converges to g in X then $\{g_n\}$ converges to g in X_G ,
- (b) The set of bounded measurable functions is a dense subset of X ; hence, of X_G , for every G ,

- (c) For every G , uniform convergence on G implies convergence in X_G and convergence in X_G implies convergence in measure on G .

THEOREM. *If $\{f_1, f_2, \dots, f_n, \dots\}$ is complete in X and $\epsilon > 0$, there is a measurable $G \subset [a, b]$, with $m(G) > (b-a) - \epsilon$, such that $\{f_2, f_3, \dots, f_n, \dots\}$ is complete in X_G .*

PROOF. Let $\{g_1, g_2, \dots, g_n, \dots\}$ be dense in X . Since $\{f_1, f_2, \dots, f_n, \dots\}$ is complete in X , it follows from (b), (c) and

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the fact that the bounded functions are dense in M , that $\{f_1, f_2, \dots, f_m, \dots\}$ is complete in M . It follows from [1] that for every n , there is a sequence $\{\phi_1, \phi_2, \dots, \phi_m, \dots\}$ of finite linear combinations of $\{f_2, f_3, \dots, f_n, \dots\}$ which converges in measure to g_n , and so has a subsequence $\{\psi_1, \psi_2, \dots, \psi_m, \dots\}$ which converges uniformly to g_n on a measurable set G_n , with $m(G_n) > (b-a) - \epsilon/2^n$. Let $G = \bigcap_{n=1}^{\infty} G_n$. Since uniform convergence on G implies convergence in X_G by (a), and since $\{g_1, g_2, \dots, g_n, \dots\}$ is dense in X_G by (c), it follows that $\{f_2, f_3, \dots, f_n, \dots\}$ is complete in X_G .

If $X = L_2[a, b]$ then $X_G = L_2(G)$, so that we have:

COROLLARY. *If $\{f_1, f_2, \dots, f_n, \dots\}$ is complete for $L_2[a, b]$ and $\epsilon > 0$ there is a measurable $G \subset [a, b]$, $m(G) > (b-a) - \epsilon$ such that $\{f_2, f_3, \dots, f_n, \dots\}$ is complete for $L_2(G)$.*

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