A REMARK ON A DECOMPOSITION SPACE OF BING

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The following question has been raised by several authors: If X and Y are two continua whose cartesian product admits a continuous associative multiplication with identity, must X and Y both admit such multiplications? We remark here, that a certain decomposition space defined by Bing yields a negative answer.

Let $\overline{B}$ denote the space described in 8 of [1]. It is a certain upper semicontinuous decomposition of S^3 each of whose elements is an arc or a point. It is shown in [1] that $\overline{B}\times S^1$ is homeomorphic with $S^3\times S^1$. Now $S^3\times S^1$ admits the structure of a topological group. Hence we need only show that $\overline{B}$ cannot admit the structure of a topological semigroup with identity. We note first the following

LEMMA. *Let D and T be two compact connected topological semigroups with identities. If $D\times T$ is topologically a manifold then D and T are already Lie groups.*

Using coordinatewise multiplication, we see that $D\times T$ is a semigroup with identity. Then however, [3], it is well known that $D\times T$ is already a group. Since $D\times T$ can then have only one idempotent it is immediate that D has only one idempotent. But a compact semigroup with a unique idempotent which is an identity must already be a group, [2]. Hence D is a topological group and, being locally connected and finite dimensional, is a Lie group. Likewise, T is already a Lie group.

Since $\overline{B}\times S^1$ is a manifold, $S^3\times S^1$, we see that were $\overline{B}$ to be a semigroup with identity it would be a Lie group. Since $\overline{B}$ is not a manifold the result follows.

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