$F(1) = 1$	$F(5) = 88$	$F(9) = 1{,}097{,}780{,}312$
$F(2) = 1$	$F(6) = 1{,}802$	$F(10) = 376{,}516{,}036{,}188$
$F(3) = 2$	$F(7) = 75{,}598$	
$F(4) = 9$	$F(8) = 6{,}421{,}599$	

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A NOTE ON THE GREATEST CROSSNORM

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Schatten has shown [**5**, Lemma 2, p. 323; **6**, Lemma 3.7, p. 55] that, if $\mathfrak{M}$ is a closed subspace of a Banach space $\mathfrak{B}$, and there is a projection of $\mathfrak{B}$ onto $\mathfrak{M}$ with bound unity, then the greatest crossnorm on the tensor product $\mathfrak{B}\odot\mathfrak{N}$ is an extension of the greatest crossnorm on $\mathfrak{M}\odot\mathfrak{N}$ for any Banach space $\mathfrak{N}$.

Now it is known that there is a projection with bound unity of the second conjugate $\mathfrak{B}^{**}$ of a Banach space $\mathfrak{B}$ onto $\mathfrak{B}_0$ (the canonical image of $\mathfrak{B}$ in $\mathfrak{B}^{**}$) for conjugate spaces $\mathfrak{B}$ and for some others [**3**, p. 580], though not for *all* Banach spaces (cf. [**7**]). For such spaces, then, the greatest crossnorm on $\mathfrak{B}^{**}\odot\mathfrak{N}$ is an extension of the greatest crossnorm on $\mathfrak{B}_0\odot\mathfrak{N}$. The purpose of this note is to show that the restriction to such spaces is unnecessary. (N.B. $\mathfrak{B}$ is sometimes embedded in $\mathfrak{B}^{**}$ by identifying it with $\mathfrak{B}_0$.)

THEOREM. *Let $\mathfrak{B}$ and $\mathfrak{N}$ be any Banach spaces. Then the greatest crossnorm on $\mathfrak{B}^{**}\odot\mathfrak{N}$ is an extension of the greatest crossnorm on $\mathfrak{B}_0\odot\mathfrak{N}$ (where $\mathfrak{B}_0$ is the canonical image of $\mathfrak{B}$ in $\mathfrak{B}^{**}$).*

Let $\mathfrak{X}$ be any element of $\mathfrak{B}_0\odot\mathfrak{N}\subset\mathfrak{B}^{**}\odot\mathfrak{N}$. Clearly (in the notation of [**2**, §2.4, pp. 347–351])

$$\gamma\{\mathfrak{B}^{**}\odot\mathfrak{N}\}(\mathfrak{X}) \leqq \gamma\{\mathfrak{B}_0\odot\mathfrak{N}\}(\mathfrak{X})$$

(since the infimum on the left-hand side is taken over a larger collection of expressions). On the other hand, there exists a continuous

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linear functional $\mathfrak{F}$ over $\mathfrak{B}_0 \odot_\gamma \mathfrak{R}$ with $\mathfrak{F}(\mathfrak{X}) = \gamma\{\mathfrak{B}_0 \odot \mathfrak{R}\}(\mathfrak{X})$ and $\|\mathfrak{F}\| = 1$ [1, Theorem 2.9.3, p. 19]. Now $\mathfrak{F}$ can be associated (cf. [4, Theorem 1.2, p. 78; 6, Theorem 3.2, p. 47]) with a continuous linear operator T on $\mathfrak{R}$ into $\mathfrak{B}^*$ with the same norm as $\mathfrak{F}$ by the rule

$$\mathfrak{F}(\bar{x} \otimes y) = (Ty)(x) \qquad (x \in \mathfrak{B},\ y \in \mathfrak{R}),$$

where $\bar{x}$ is the canonical image of x in $\mathfrak{B}^{**}$. We now construct a continuous linear operator T' on $\mathfrak{R}$ into $\mathfrak{B}^{***}$ by defining $T'y$ to be the canonical image of Ty in $\mathfrak{B}^{***}$ for each y of $\mathfrak{R}$. This is associated with a continuous linear functional $\mathfrak{F}'$ over $\mathfrak{B}^{**} \odot_\gamma \mathfrak{R}$ with the same norm as T' by the rule

$$\mathfrak{F}'(X \otimes y) = (T'y)(X) \qquad (X \in \mathfrak{B}^{**},\ y \in \mathfrak{R}).$$

Then

$$\mathfrak{F}'(\bar{x} \otimes y) = (T'y)(\bar{x}) = \bar{x}(Ty) = (Ty)(x) = \mathfrak{F}(\bar{x} \otimes y),$$

and so $\mathfrak{F}'$ is an extension of $\mathfrak{F}$, and

$$\|\mathfrak{F}'\| = \|T'\| = \|T\| = \|\mathfrak{F}\| = 1.$$

Hence

$$\gamma\{\mathfrak{B}_0 \odot \mathfrak{R}\}(\mathfrak{X}) = |\mathfrak{F}(\mathfrak{X})| = |\mathfrak{F}'(\mathfrak{X})| \leqq \gamma\{\mathfrak{B}^{**} \odot \mathfrak{R}\}(\mathfrak{X}).$$

This inequality, in conjunction with that above, shows that

$$\gamma\{\mathfrak{B}^{**} \odot \mathfrak{R}\}(\mathfrak{X}) = \gamma\{\mathfrak{B}_0 \odot \mathfrak{R}\}(\mathfrak{X}).$$

Since the element $\mathfrak{X}$ of $\mathfrak{B}_0 \odot \mathfrak{R}$ was arbitrary, this completes the proof of the theorem.

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