A THEOREM ON GENERIC NORMS OF STRICTLY POWER ASSOCIATIVE ALGEBRAS

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Let A be a finite-dimensional strictly power associative algebra with an identity element over an arbitrary field k, and let

$$m(x) = x^n + \sum_{i=1}^{n} \lambda_i(x) x^{n-i}$$

be its generic minimum polynomial [1]. The coefficients λ_i are polynomial functions on A. Such a function f is called a Lie invariant under a linear transformation L of the underlying vector space if $f(a+tL(a)) \equiv f(a) \pmod{t^2}$ where t is an indeterminate and f is extended in the usual way to the vector space over $k(t)$; in particular, if f is a linear form on A (for instance the generic trace λ_1), this means that $f(L(a))=0$.

THEOREM. *The coefficients λ_i of the generic minimum polynomial are Lie invariant under every derivation d of A.*

Assuming that A is a Jordan algebra (over a field of characteristic not two), that $i=1$ and that d is the inner derivation which sends a into $b \cdot ac - ba \cdot c$, we have the

COROLLARY. *The identity $\lambda_1(b \cdot ac) = \lambda_1(ba \cdot c)$ holds in any Jordan algebra.*

This result has been obtained independently by N. Jacobson (unpublished).

PROOF OF THE THEOREM. Let K be an arbitrary extension of k. The extensions of the forms λ_i and of the derivation d to A_K will be denoted by the same symbols λ_i and d. Let t be an indeterminate scalar and, for $a, b \in A_K$, denote by $\{a, b\}_i$ (resp. $\mu_i(a, b)$) the coefficient of t in $(a+tb)^i$ (resp. in $\lambda_i(a+tb)$). As $m(a+tb)$ vanishes identically, the coefficient of t in it must be zero, that is,

$$(1) \qquad \{a, b\}_n + \sum_{i=1}^{n} \lambda_i(a) \cdot \{a, b\}_{n-i} + \sum_{i=1}^{n} \mu_i(a, b) \cdot a^{n-i} = 0$$

It is easily seen that $d(a^i) = \{a, d(a)\}_i$; therefore

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$$d(m(a)) = \{a, d(a)\}_n + \sum_{i=1}^{n} \lambda_i(a) \cdot \{a, d(a)\}_{n-i} = 0 \tag{2}$$

for every $a \in A_K$. Setting $b = d(a)$ in (1) and subtracting (2), we have

$$\sum_{i=1}^{n} \mu_i(a, d(a)) \cdot a^{n-i} = 0.$$

If a is generic (over k), it does not satisfy any polynomial identity of order $n-1$, with coefficients in K; thus

$$\mu_i(a, d(a)) = 0,$$

and the same relations then hold for arbitrary $a \in A_K$.

By the definition of the $\mu_i(a, b)$ this is the Lie invariance of the $\lambda_i(a)$ which we wished to prove.

BIBLIOGRAPHY

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