fact that a Friedberg numbering τ does not satisfy the recursion theorem relative to τ (footnote 2) may provide some limitation on the usefulness of replacing Gödel numberings by Friedberg numberings in certain technical investigations.

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THE CLOSED CONVEX HULL OF CERTAIN EXTREME POINTS

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Let H be a Hausdorff space and let $C(H)$ be the space of bounded continuous real-valued functions on H , $C(H)$ having the usual supremum norm. Also, let S be the unit ball of $C(H)$, let E be the set of extreme points of S , and let K be the set of characteristic functions of open-and-closed sets in H .

It is known [1, §3, p. 10] that when H is compact, S is the norm closed convex hull of E if and only if H is totally disconnected. It is known also [4, p. 103] that if H is extremally disconnected, then S is the norm closed convex hull of E . The purpose of this paper is to give a necessary and sufficient condition for S to be the norm closed convex hull of E . The theorem for compact H mentioned above is a special case of our result (cf. [3, p. 247]). We are indebted to the referee for observing that if H is completely regular, our result follows at once from the compact case (cf. [3, p. 88]). Our condition is also necessary and sufficient for each of the subsets E and K to be fundamental (cf. [2, p. 35]) in $C(H)$.

If f is a function in $C(H)$, we shall call the set

$$Z(f) = \{h: h \in H, f(h) = 0\}$$

the zero-set of the function f . We observe that if r is a real number, then the sets $\{h: f(h) = r\}$, $\{h: f(h) \geq r\}$, and $\{h: f(h) \leq r\}$ are zero-

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Let S be the closed convex hull of E and let $Z(f)$ and $Z(g)$ be disjoint zero-sets in H . The function F defined by the relation $F(h) = |f(h)| / (|f(h)| + |g(h)|)$ for all h in H has the properties $0 \leq F(h) \leq 1$ for h in H , $F(h) = 0$ for h in $Z(f)$, and $F(h) = 1$ for h in $Z(g)$. Since F is in S , there is a function G in the convex hull of E such that $\|F - G\| < 1/2$. Since G assumes only a finite number of distinct values on H , the sets $H_1 = \{h: G(h) \leq 1/2\}$ and $H_2 = \{h: G(h) > 1/2\}$ are disjoint open-and-closed sets containing $Z(f)$ and $Z(g)$, respectively. Since $H_1 \cup H_2 = H$, we see that H has the zero-set separation property. This completes the proof.

COROLLARY 1. *The set E of extreme points of the unit ball S in $C(H)$ is fundamental in $C(H)$ if and only if H has the zero-set separation property.*

PROOF. Let F be a nonzero element in $C(H)$ and let $\epsilon > 0$ be assigned. If H has the zero-set separation property, by the theorem there is a function g in the convex hull of E such that $\|(F/\|F\|) - g\| < \epsilon/\|F\|$. It follows that $\|F - (\|F\|g)\| < \epsilon$ and, hence, that E is fundamental in $C(H)$.

If the set E of extreme points of S is fundamental in $C(H)$, H can be shown to have the zero-set separation property by an argument similar to an argument used in the proof of the above theorem. We omit the details.

COROLLARY 2. *The set K of all characteristic functions of open-and-closed subsets of H is fundamental in $C(H)$ if and only if H has the zero-set separation property.*

PROOF. If we note that the previously constructed function g_N is a linear combination of elements in K , the proof follows easily. The details of the proof are left to the reader.

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