A TOTALLY NONAPOSYNDETIC, COMPACT, HAUSDORFF SPACE WITH NO CUT POINT

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This paper gives a negative answer to the following question posed by A. D. Wallace. Does each totally nonaposyndetic,[2] bicompact, connected, Hausdorff space contain a weak cut point?[3] Conditions which *assure* the existence of weak cut points in totally nonaposyndetic continua are given in [1] and [2].

Example 1 gives a totally disconnected, perfect, first-countable, bicompact, Hausdorff space K, of cardinal c, such that (the closure of) each subset of cardinal less than c is nowhere dense. The space K will be used in describing the main example.

EXAMPLE 1. Let I be the space of all functions f from the natural numbers into $[0, 1]$. Let I have the order topology relative to the lexicographical order.[4] Let K be the set of all f in I such that, if $f(i)=\frac{1}{2}$, then either $f(j)=0$, for $j>i$, or $f(j)=1$, for $j>i$.

EXAMPLE 2. Let z be the first ordinal that is preceded by c ordinals. Let S consist of four long lines (of "length" c), put together as in Figure 1, with the connecting points a_z and a_z'.

Let K' be a copy of K and, for each point or subset A of K, let A' denote the corresponding point or subset of K'. Let $k_1, k_2, \cdots, k_x, \cdots$, for $x<z$, be a most economical well-ordering of K. Let K'' be the union of K and K' with k_1 and k_1' identified. For $y<z$, let K_y be the closure of $\{k_x \mid x \leqq y\}$. Let $K_z = K$.

Let H be the decomposition space of $K'' \times S$, the nondegenerate elements of which are $\{(k, a_y) \mid k \in K_y\}$ and $\{(k', a_y') \mid k' \in K_y'\}$, for $y \leqq z$.

Figure 2 will aid in visualizing H. It does not indicate the identifications that prevent cutting by the vertices. The intervals in the figure represent copies of S.

H is a totally nonaposyndetic, bicompact, Hausdorff space that

Presented to the Society, April 14, 1962; received by the editors February 13, 1962 and, in revised form, January 4, 1963.

[1] This work was completed while the author was on leave from Emory University as an N.S.F. Science Faculty Fellow at the University of Wisconsin.

[2] A continuum T is *totally nonaposyndetic* if it is nonaposyndetic at each point, i.e., if for each point x in T there is a point y in T such that each subcontinuum of T that contains x in its interior also contains y.

[3] The author wishes to thank Professor F. Burton Jones for calling this question to his attention.

[4] $g<f$ if, for some i, $g(i)<f(i)$ and $g(j)=f(j)$ for $j<i$.

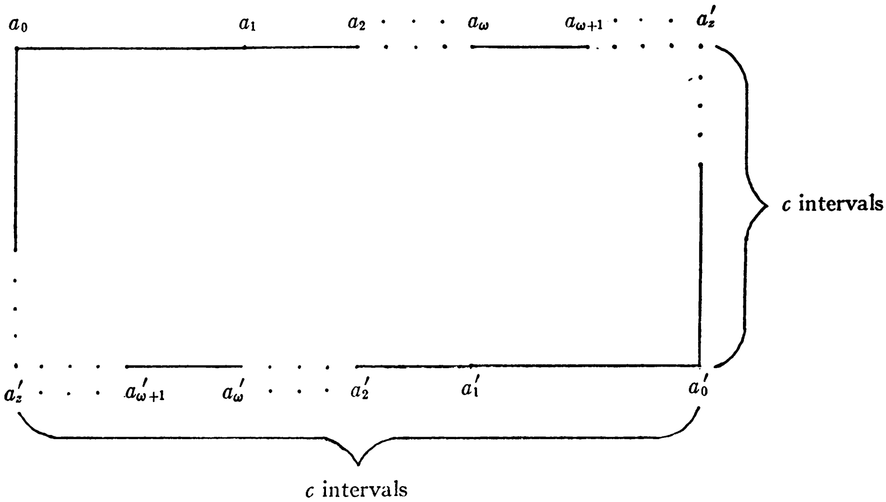

FIGURE 1

contains no weak cut point. H satisfies the first axiom of countability at each point of a dense open set.

The total nonaposyndesis of H can be seen as follows. Let v be the vertex $\{(k, a_z) | k \in K_z\}$ of H. If $p \neq v$ is a point of the part of H

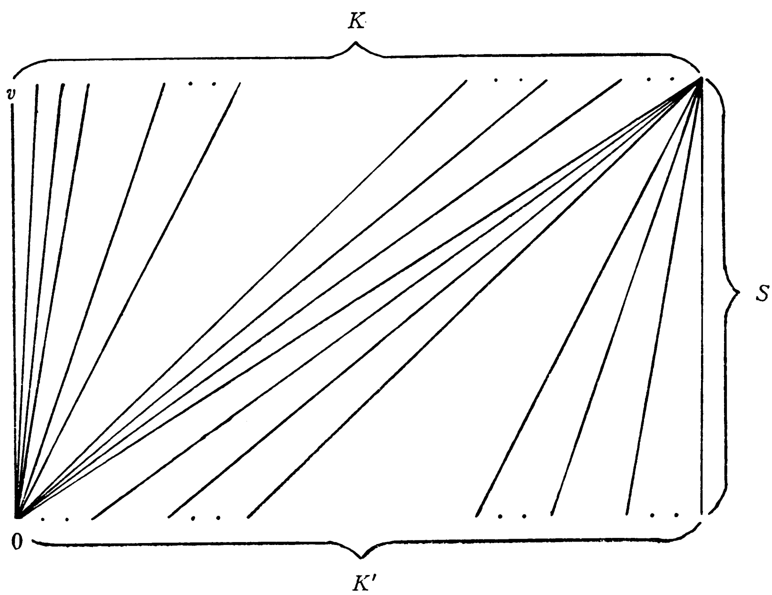

FIGURE 2

that comes from $K \times S$ and D is an open set in $H-p$ containing v, then p is not an interior point of the p-component of $H-D$. This follows from the fact that K_y is nowhere dense in K, for $y<z$. It implies that H is nonaposyndetic at p with respect to v. As a consequence of this and the corresponding fact about the part of H that comes from $K' \times S$, the continuum H is totally nonaposyndetic.

QUESTION. Does each totally nonaposyndetic, bicompact, connected, Hausdorff space, that satisfies the first axiom of countability, contain a weak cut point?

REFERENCES

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