AN EXAMPLE IN ČECH COHOMOLOGY

DANIEL S. KAHN[1]

In this note, we give an example of a compact space X with integral Čech cohomology groups $H^q(X)=0$, $q>0$, but which can be mapped essentially onto the three-sphere S^3. This cannot occur for finite-dimensional X [2].

We construct such an X for each odd prime p, which we now suppose fixed. Define X_0 to be $S^{2p}\cup_p e^{2p+1}$, the $2p$-sphere with a $(2p+1)$-cell attached by a map of degree p. Inductively, we define X_{n+1} to be the $(2p-2)$-fold suspension $E^{2p-2}X_n$ of X_n, $n\geqq 0$. We also define maps $\alpha_n\colon X_n\to X_{n-1}$, $n>1$, by $\alpha_n=E^{2p-2}\alpha_{n-1}$, where α_1 is defined as follows: Let $\beta\colon S^{2p}\to S^3$ represent a generator of $\pi_{2p}(S^3;p)\approx Z_p$. Then $E^{2p-3}\beta\colon S^{4p-3}\to S^{2p}$ admits a coextension $\beta'\colon S^{4p-2}\to S^{2p}\cup_p e^{2p+1}$ [**4**, p. 13]. Since the homotopy class of β' is of order p, β' admits an extension $\alpha_1\colon S^{4p-2}\cup_p e^{4p-1}\to S^{4p-1}\cup_p e^{2p+1}$. We note that β admits an extension $\alpha\colon S^{2p}\cup_p e^{2p+1}\to S^3$. We now define $X=\operatorname{Inv\,Lim}\{X_n,\alpha_n\}$. It is evident that $H^q(X)=0$, $q>0$. The composites $f_n=\alpha\alpha_1\alpha_2\cdots\alpha_n\colon X_n\to S^3$ define a map $f\colon X\to S^3$. The proof that f is essential depends on the following result of Toda [**5**], [**1**].

THEOREM [TODA]. *Each f_n is an essential map. Further, all suspensions of f_n are essential.*

Since $[X, S^3]$, the set of homotopy classes of maps of $X\to S^3$, is equal to $\operatorname{Dir\,Lim}\{[X_n,S^3],\alpha_{n+1}^*\}$ [**3**, p. 228], f is essential.

X has the further property that $E^{n(2p-2)}X=X$, $n>0$. The theorem of Toda implies that each $E^{n(2p-2)}f\colon E^{n(2p-2)}X=X\to S^{3+n(2p-2)}$ is also essential.

BIBLIOGRAPHY

1. J. F. Adams, *Lectures on $K^*(X)$*, Mimeographed notes, University of Manchester, 1962.
2. S. T. Hu, *Mappings of a normal space into an absolute neighborhood retract*, Trans. Amer. Math. Soc. **64** (1948), 336–358.
3. E. Spanier, *Borsuk's cohomotopy groups*, Ann. of Math. **50** (1949), 203–245.
4. H. Toda, *Composition methods in homotopy groups of spheres*, Princeton Univ. Press, Princeton, N. J., 1962.
5. ———, *On unstable homotopy of spheres and classical groups*, Proc. Nat. Acad. Sci. **46** (1960), 1102–1105.

THE UNIVERSITY OF CHICAGO

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