

PROCEEDINGS  
OF THE  
AMERICAN MATHEMATICAL SOCIETY

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## Journals Published by the American Mathematical Society

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### Bulletin of the American Mathematical Society

This journal is the official organ of the Society. It reports official acts of the Society and the details of its meetings. It contains some of the officially invited addresses presented before the Society, reviews of advanced mathematical books, research problems and a department of research announcements.

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## Mathematical Reviews

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## Soviet Mathematics—Doklady

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This journal announces the programs of the meetings of the Society. It carries the abstracts of all contributed papers presented at the meetings of the Society and publishes news items of interest to mathematical scientists.

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A journal devoted to original papers in numerical analysis, the application of numerical methods and high-speed calculator devices, the computation of mathematical tables, the theory of high-speed calculating devices and other aids to computation. In addition it publishes reviews and notes in these and related fields.

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## Contents—Continued from back cover

|   |      |
|---|------|
| Prime rings with maximal annihilator and maximal complement right ideals. By Kwangil KOH and A. C. MEWBORN..... | 1073 |
| Analytic functions in Banach spaces. By E. F. WHITTLESEY.....   | 1077 |
| Maximal quotient rings. By CARL FAITH and YUZO UTUMI.....   | 1084 |
| The Schnirelmann density of the $k$ -free integers. By R. L. DUNCAN.....  | 1090 |
| On a problem connected with the Vandermonde determinant. By J. KIEFER and J. WOLFOWITZ.....                     | 1092 |
| Reflexive semigroups. By R. O. FULP and PAUL HILL.....  | 1096 |
| Prime matrix rings. By R. E. JOHNSON.....   | 1099 |
| Powers in eighth-groups. By SEYMOUR LIPSCHUTZ.....  | 1105 |
| On a class of Lie algebras. By R. H. OEHMKE.....  | 1107 |
| Commutative algebras satisfying an identity of degree four. By J. M. OSBORN.....                                | 1114 |
| Some properties of localization and normalization. By JOSEPH LIPMAN.....  | 1120 |
| Symmetry for the enveloping algebra of a restricted Lie algebra. By J. R. SCHUE.....                            | 1123 |
| Representations of Lie algebras by normal operators. By THOMAS SHERMAN.....                                     | 1125 |

## SHORTER NOTES

|   |      |
|---|------|
| A note on a theorem of I. Niven. By W. J. LEAHEY.....   | 1130 |
| A matrix version of Rennie's generalization of Kantorovich's inequality. By B. MOND.....        | 1131 |
| Note on a property of the elementary symmetric functions. By J. N. DARROCH and JANE PITMAN..... | 1132 |
| A note on addition chains. By C. T. WHYBURN.....  | 1134 |

# CONTENTS

Vol. 16, No. 5

OCTOBER, 1965

Whole No. 95

|  | Page |
|--|------|
| Radii of star-likeness and close-to-convexity. By M. S. ROBERTSON.....   | 847  |
| Lower bounds for solutions of hyperbolic inequalities. By HAJIMU OGAWA.....  | 853  |
| Partial differential equations and difference equations. By LEOPOLD FLATTO.....  | 858  |
| Harnack's and Weyl's inequalities. By MARVIN MARCUS.....   | 864  |
| $U(\epsilon)$ -sets for Walsh series. By V. L. SHAPIRO.....  | 867  |
| Pairs of Carleman-type integral equations. By R. C. MAC CAMY.....  | 871  |
| A remark on an arithmetic theorem of Chevalley. By H. BASS.....  | 875  |
| On a class of holomorphic functions. By NICOLAS ARTÉMIADIS.....  | 879  |
| Comparison theorems for elliptic differential equations. By COLIN CLARK and C. A. SWANSON.....                           | 886  |
| Local trajectory equivalence of differential systems. By COURTNEY COLEMAN.....   | 890  |
| On the inversion of matrices and linear operators. By W. V. PETRYSHYN.....   | 893  |
| On differential equations of mode 2. By J. C. C. NITSCHÉ.....  | 902  |
| An abstract formulation of some theorems on cluster sets. By ULYSSES HUNTER.....   | 909  |
| On the lifting property. By W. F. DONOGHUE, JR.....  | 913  |
| Inequalities and a limit theorem for certain Wiener integrals. By J. YEH.....  | 915  |
| The estimation of some determinants of Toeplitz-type. By H. O. POLLAK and L. A. SHEPP.....                               | 919  |
| A note on indefinite integrals. By M. H. TAIBLESON.....  | 923  |
| On a note of Marcus concerning a problem posed by Frink. By R. B. DARST and R. E. ZINK.....                              | 926  |
| Convergence of closed subsets in a topological space. By E. G. EFFROS.....   | 929  |
| On the continuous function space of a basically disconnected space. By D. B. GOODNER.....                                | 932  |
| Stationary sets and determining sets for certain classes of Darboux functions. By A. M. BRUCKNER and J. LEONARD.....     | 935  |
| The Shilov boundary of the algebra of measures on a group. By J. L. TAYLOR.....  | 941  |
| A representation theorem for a continuous linear transformation on a space of continuous functions. By D. H. TUCKER..... | 946  |
| Counterexample to the Morse-Sard theorem in the case of infinite-dimensional manifolds. By IVAN KUPKA.....               | 954  |
| The degree of approximation by linear operators. By P. C. CURTIS, JR.....  | 957  |
| Ergodic and spectral analysis of certain infinite measure preserving transformations. By WILLIAM PARRY.....              | 960  |
| A Choquet boundary for the product of two compact spaces. By M. W. GROSSMAN.....   | 967  |
| On a problem of G. E. Sacks. By A. H. LACHLAN.....   | 972  |
| The spectrum of an operator in Banach space. By M. D. GEORGE.....  | 980  |
| An invariance theorem for representations of Banach algebras. By L. T. GARDNER.....                                      | 983  |
| On the existence of [Schauder] decompositions in Banach spaces. By B. L. SANDERS.....                                    | 987  |
| An approximation theorem for a class of operators. By G. K. LEAF.....  | 991  |
| A note on normal dilations. By ARNOLD LEBOW.....   | 995  |
| Minkowskian distribution of discs. By L. FEJES TÓTH.....   | 999  |
| Homology of deleted products in dimension one. By A. H. COPELAND, JR.....  | 1005 |
| On the combinatorial Schoenflies conjecture. By ERHARD LUFT.....   | 1008 |
| A note on the parallelizability of real Stiefel manifolds. By DAVID HANDEL.....  | 1012 |
| A compact convex set in $E^3$ whose exposed points are of the first category. By H. H. CORSON.....                       | 1015 |
| A theorem on homotopically equivalent $(2k+1)$ -manifolds. By Y.-F. WONG.....  | 1022 |
| Embedding products of chainable continua. By RALPH BENNETT.....  | 1026 |
| Nondifferentiability of retractions of $C^*$ to subvarieties. By M. GILMARTIN.....                                       | 1028 |
| On piecewise linear immersions. By M. W. HIRSCH.....   | 1029 |
| A characterization of the almost periodic homeomorphisms on the closed 2-cell. By N. E. FOLAND.....                      | 1031 |
| On points of Jacobian rank $k$ . II. By P. T. CHURCH.....  | 1035 |
| Parallel umbilical hypersurfaces. By C. M. FULTON.....   | 1039 |
| The ring $C(X)$ determines the category of $X$ . By W. W. COMFORT and STELIOS NEGREPONTIS.....                           | 1041 |
| A note on holonomy. By A. P. WHITMAN and LAWRENCE CONLON.....  | 1046 |
| On the center of the group of a link. By KUNIO MURASUGI.....   | 1052 |
| Extreme Hamiltonian circuits. Resolution of the convex-even case. By L. V. QUINTAS and FRED SUPNICK.....                 | 1058 |
| On the invariants of a vector subspace of a vector space over a field of characteristic two. By VERA PLESS.....          | 1062 |
| Characterization of totally unimodular matrices. By PAUL CAMION.....   | 1068 |

*Continued on inside back cover*