## SHORTER NOTES

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# EXAMPLES OF MINIMAL PARALLEL SLIT DOMAINS

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We call a plane domain $D$ ($\infty \in D$) a *minimal* horizontal parallel slit domain if $\operatorname{Re} a_1 \leqq 0$ for every meromorphic univalent function in $D$ having expansion $z + a_1/z + \cdots$ about $\infty$. It is a horizontal parallel slit domain, i.e., a domain each boundary component of which is a point or a line segment parallel to the real axis. However, a horizontal parallel slit domain is not always minimal. Certain criteria for minimality have been obtained by Grötzsch [**2**, p. 188] and Jenkins [**3**, pp. 81–85]. From them it is seen that, if the projection of the complement of $D$ on the imaginary axis has vanishing linear measure, then $D$ is minimal. This is not a necessary condition; moreover, Grötzsch constructed an example of a minimal horizontal slit domain the projection of whose complement on the imaginary axis is an interval [**3**, p. 198]. We shall exhibit in Theorem 1 a simpler construction of a domain with the same nature.

A compact set $E$ is said to be of class $N_D$ if its complementary domain does not carry nonconstant analytic functions with finite Dirichlet integral. A compact set $E$ is of class $N_D$ if and only if its complementary domain is a minimal horizontal and vertical parallel slit domain (see Ahlfors-Beurling [**1**, pp. 109–112]). As a consequence, $E$ is of class $N_D$ if its projections on the real and imaginary axes have vanishing linear measure. Again this condition is far from being necessary; in fact, the above mentioned example of Grötzsch is, as is easily seen, of class $N_D$. We shall, moreover, exhibit an example of a set of class $N_D$ whose projection on any line is an interval (Theorem 2).

THEOREM 1. *There exists a minimal horizontal parallel slit domain the projection of whose complement on the imaginary axis is an interval.*

PROOF. Consider $E_1 = e \times e$ where $e$ is Cantor's ternary set. It is of class $N_D$ since the ternary set has vanishing linear measure. Rotate it by 45° about the "center" of $E_1$, and let the resulting set be $E_2$. Then the complementary domain of $E_2$ is an example of the desired kind.

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THEOREM 2. *There exists a set of class $N_D$ whose projection on any line is an interval.*

PROOF. Retain the above notations. $E = E_1 \cup E_2$ serves.

## REFERENCES

1. L. A. Ahlfors and A. Beurling, *Conformal invariants and function-theoretic null-sets*, Acta Math. **83** (1950), 101–129.

2. H. Grötzsch, *Zum Parallelschlitztheorem der konformen Abbildung schlichter unendlich-vielfach zusammenhängender Bereiche*, Leipziger Berichte **83** (1931), 185–200.

3. J. A. Jenkins, *Univalent functions and conformal mapping*, Springer, Berlin, 1958.

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