the above mentioned theorem of Cartan and Carathéodory the proof is finished.

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A CHARACTERIZATION OF TAME 2-SPHERES IN E^3

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In this note, the tame 2-spheres in E^3 are characterized partly in terms of homology and the arcs they contain. In a similar way, the compact 2-manifolds with boundary are characterized. If K is a finite topological 2-complex in E^3 and v is a vertex of K, then $\mathrm{St}\, v$ is the star of v, $\dot{\mathrm{S}}\mathrm{t}\, v$ is the open star of v, and $\mathrm{Lk}\, v = \mathrm{St}\, v - \dot{\mathrm{S}}\mathrm{t}\, v$ is the link of v. The trivial 1-dimensional homology group of K will be denoted by $H_1(K) = 0$.

An n-manifold with boundary is a separable metric space such that each point has a neighborhood whose closure is topologically equivalent to a closed n-cell.

Theorem 1. *Let K be a finite topological 2-complex in E^3 such that*
(i) *K is connected,*
(ii) *$\mathrm{Lk}\, v$ is connected for each vertex v in K,*
(iii) *$H_1(K) = 0$, and*
(iv) *K contains only tame arcs.*
Then K is either a disk or a 2-sphere.

Proof. Since K contains no wild arcs and $\mathrm{Lk}\, v$ is connected, each 1-simplex in K lies on exactly one or two 2-simplices in K [2]. Since

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[1] These results form a part of the author's doctoral dissertation written at the Virginia Polytechnic Institute in 1964 under the direction of Professor P. H. Doyle.

Lk v is connected, Lk $v=G$, a connected graph. Again by (iv), no vertex of G is of order greater than two, and so G is a 1-manifold with boundary. Thus G is either an arc or a simple closed curve. So $\dot{\text{S}}\text{t}\ v$ is a 2-manifold with boundary, and hence K is a 2-manifold with boundary. By (iii), K is either a disk or a 2-sphere.

COROLLARY 2. *If K satisfies the conditions of the Theorem and if, in addition, no arc in K separates K, then K is a 2-sphere.*

By the addition of one more condition to Corollary 2, we obtain a characterization of tame 2-spheres in E^3.

THEOREM 3. *A necessary and sufficient condition that a finite connected topological 2-complex K in E^3 is a tame 2-sphere is that K satisfies the following conditions:*

(i) Lk v *is connected for each vertex v in K,*
(ii) $H_1(K)=0$,
(iii) *K contains only tame arcs,*
(iv) *No arc in K separates K, and*
(v) *E^3-K is locally simply connected at each point of K.*

PROOF. By Corollary 2, K is a 2-sphere, and by Bing [1], condition (v) insures that K is tame. Conversely, it is clear that a tame 2-sphere satisfies the conditions.

If the requirement that $H_1(K)=0$ is omitted in Theorem 1, we obtain the following corollary to the proof of Theorem 1.

THEOREM 4. *A finite topological 2-complex K in E^3 is a compact 2-manifold with boundary if and only if K satisfies the following conditions:*

(i) *K is connected,*
(ii) *Lk v is connected for each vertex v in K, and,*
(iii) *K contains no wild arcs.*

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