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CORRECTION TO “ON MATRICES WHOSE REAL LINEAR COMBINATIONS ARE NONSINGULAR”

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We are grateful to Professor B. Eckmann for pointing out an error in the proof of Lemma 4(b) of our paper [1]. This error invalidates Lemma 4(b) and that part of Theorem 1 which states the values of $Q(n)$, $Q_H(n)$. The error occurs immediately after the words “arguing as is usual for the complex case, we find”; it consists in manipulating as if the ground field Λ were commutative.

The proof of Lemma 4(b) can be repaired, as will be shown below, but it leads to a different conclusion from that given. Our paper should therefore be corrected as follows.

- (i) In Theorem 1, the values of $Q(n)$ and $Q_H(n)$ should read

$$Q(n) = \rho(\tfrac{1}{2}n) + 4, \quad Q_H(n) = \rho(\tfrac{1}{4}n) + 5."$$

The two sentence paragraph following Theorem 1 should be deleted. It remains interesting to ask what topological phenomena (if any) can be related to our algebraic results.

- (ii) In Lemma 4, part (b) should read

$$Q_H(n) + 3 \leq R(4n)."$$

The proof is as follows.

Let W be a k -dimensional space of $n \times n$ Hermitian matrices with entries from Q which has the property P . The space Q^n is a real vector space of dimension $4n$. For each $A \in W$ and each pure imaginary $\mu \in Q$ we consider the following real-linear transformation from Q^n to itself:

$$B(x) = Ax + x\mu.$$

We claim that the $(k+3)$ -dimensional space formed by such B has the property P . For suppose that such a B is singular; then there is a nonzero x such that

$$Ax = -x\mu;$$

then we have

$$x^*(Ax) = -x^*x\mu,$$

$$(x^*A)x = (-x\mu)^*x = \mu x^*x.$$

Since x^*x is real and nonzero, we have $\mu=0$; hence A is singular and $A=0$. This completes the proof.

(iii) In Lemma 5, there should be added a second part, reading

$$“(b) \quad R_H(n) + 3 \leq Q(n).”$$

PROOF. Let W be a k -dimensional space of $n \times n$ real symmetric matrices which has the property P . Consider the matrices

$$A + \mu I,$$

where A runs over W and μ runs over the pure imaginary elements of Q . We claim that they form a space of dimension $k+3$ with the property P . In fact, suppose that such a matrix is singular; and suppose to begin with, that μ is nonzero. Then the elements $1, \mu$ form an R -base for a subalgebra of Q which we may identify with C . Choose a C -base of Q ; this splits Q^n as the direct sum of two copies of C^n . Since the matrix $A + \mu I$ acts on each summand, it must be singular on at least one. That is, the real symmetric matrix A has a nonzero complex eigenvalue which is purely imaginary, a contradiction. Hence μ must be zero and $B=A$. Now choose an R -base of Q ; this splits Q^n as the direct sum of 4 copies of R^n . Since A acts on each summand, it must be singular on at least one. That is, A must be singular; hence $A=0$. This completes the proof.

(iv) The final paragraph of the paper should be deleted, and replaced by the following proof.

“Finally, Lemmas 5(b), 3 and 4(b) show that

$$Q(n) - R_H(n) \geq 3,$$

$$Q_H(2n) - Q(n) \geq 1,$$

$$R(8n) - Q_H(2n) \geq 3.$$

But we have already shown that

$$R(8n) - R_H(n) = 7,$$

so all these inequalities are equalities. This completes the proof of Theorem 1."

We note that this method provides an alternative proof of Lemma 5 ($R(8n) - R_H(n) \geq 7$), without using the Cayley numbers.

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