SHORTER NOTES

The purpose of this department is to publish very short papers of an unusually elegant and polished character, for which there is no other outlet.

A SHORT AND ELEMENTARY PROOF THAT A PRODUCT OF SPHERES IS PARALLELIZABLE IF ONE OF THEM IS ODD

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Kervaire [1] has shown that the product of several spheres, one of which is of odd dimension, is a parallelizable manifold. This note observes a short and elementary proof of this fact. The question does not arise if the spheres are all even dimensional, for the Euler characteristic shows that there is not even a nonzero vector field.

First observe that the tangent bundle of a product manifold, $\tau(M_1 \times M_2)$, is canonically isomorphic to the Whitney sum $p_1^{-1}\tau M_1 \oplus p_2^{-1}\tau M_2$ where $p_i^{-1}\tau M_i$ is the bundle induced by p_i: $M_1 \times M_2 \to M_i$ (the projection) from τM_i $(i=1, 2)$. Secondly, if M_1, M_2 are s-parallelizable (i.e. $\xi \oplus \tau M_i \cong \xi^{m_i+1}$ where ξ is a trivial line bundle and ξ^m is the m-fold Whitney sum of ξ with itself) then $M_1 \times M_2$ is s-parallelizable. This follows from:

$$\begin{aligned} \xi \oplus \tau(M_1 \times M_2) &\cong \xi \oplus p_1^{-1}\tau M_1 \oplus p_2^{-1}\tau M_2 \cong p_1^{-1}(\xi \oplus \tau M_1) \oplus p_2^{-1}(\tau M_2) \\ &\cong \xi^{m_1+1} \oplus p_2^{-1}\tau M_2 \cong \xi^{m_1} \oplus p_2^{-1}(\xi \oplus \tau M_2) \\ &\cong \xi^{m_1} \oplus \xi^{m_2+1} \cong \xi^{m_1+m_2+1} \end{aligned}$$

where dimension $M_i = m_i > 0$.

Now let the Euler characteristic of M_1 be zero; there is a nonzero vector field on M_1 and the tangent bundle is the Whitney sum of some (m_1-1)-plane bundle α with ξ. $M_1 \times M_2$ is parallelizable for we have

$$\begin{aligned} \tau(M_1 \times M_2) &\cong p_1^{-1}\tau M_1 \oplus p_2^{-1}\tau M_2 \cong p_1^{-1}(\alpha \oplus \xi) \oplus p_2^{-1}\tau M_2 \\ &\cong (p_1^{-1}\alpha) \oplus \xi \oplus p_2^{-1}\tau M_2 \cong p_1^{-1}\alpha \oplus p_2^{-1}(\xi \oplus \tau M_2) \\ &\cong p_1^{-1}\alpha \oplus \xi^{m_2+1} \cong p_1^{-1}(\alpha \oplus \xi) \oplus \xi^{m_2} \\ &\cong p_1^{-1}(\tau M_1) \oplus \xi^{m_2} \cong p_1^{-1}(\tau M_1 \oplus \xi) \oplus \xi^{m_2-1} \\ &\cong \xi^{m_1+1} \oplus \xi^{m_2-1} \cong \xi^{m_1+m_2}. \end{aligned}$$

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The result now follows by an easy induction and observing that spheres are s-parallelizable; (the normal bundle of the usual inclusion $S^m \subseteq R^{m+1}$ is ξ and its Whitney sum with τS^m is $\tau R^{m+1} | S^m = \xi^{m+1}$) and $S^{2m+1} \subseteq R^{2m+2} = \mathbf{C}^{m+1}$ has a nonzero vector field obtained by multiplying the position vector by i.

Reference

1. M. A. Kervaire, *Courbure intégrale généralisée et homotopie*, Math. Ann. **131** (1956), 219–252; Théorème XII.

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ON CONVEX COMBINATIONS OF BLASCHKE PRODUCTS

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1. If f is analytic in the open unit disk and has modulus bounded by 1, then there is a sequence of finite Blaschke products, $\{E_n\}$, so that $E_n(z) \to f(z)$ for all $|z| < 1$ (see Caratheodory [**1**, p. 13]). The question has been raised by Phelps whether, if f is also assumed continuous on the closed disk, f can be uniformly approximated by convex combinations of finite Blaschke products (see Phelps [**5**] for the motivation of this problem). We do not solve this problem but show the approximation is possible in H_1 norm. We wish to point out that although the work of Nishiura and Waterman [**4**] leaves open the possibility of a Banach-Saks theorem in L_1, it does show that such a theorem is false in H_1.

2. First we claim if f is a sup norm 1 member of the disk algebra, we may approximate f by a polynomial which has no zeros on the boundary of the disk. This polynomial will factor into a finite Blaschke product and a nonvanishing function h. It is clear that to approximate f by a convex combination of Blaschke products it is sufficient to so approximate h. Now h has an analytic square root g with a sup norm bound 1. We apply the Caratheodory result to obtain a sequence of finite Blaschke products $\{E_n\}$ with $E_n(z) \to g(z)$ for $|z| < 1$. Now all the E_n are in H_2 and have H_2 norm 1. The compactness of the H_2 unit ball allows the extraction of a subsequence (for which we use the

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