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POSITIVE $H^{1/2}$ FUNCTIONS ARE CONSTANTS

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The Koebe function $z/(1+z)^2$ is positive everywhere on $|z|=1$, $z\neq -1$, and lies in the Hardy class H^p for every $p<1/2$. We show that this behavior is extreme by proving the following

THEOREM. *If $f(z)\in H^{1/2}$ and if $f(z)\geqq 0$ a.e. on $|z|=1$ then $f(z)$ is a constant.*

PROOF. We may assume that $f(z)$ is not identically 0. If $B(z)$ denotes the Blaschke product for the zeros of $f(z)$ then, as usual, we can write

$$f(z) = B(z)F^2(z), \qquad F(z) \in H^1. \tag{1}$$

We write the condition $f(z)\geqq 0$ as the equation $f(z)=|f(z)|$ and conclude from (1) that

$$B(z)F^2(z) = |F^2(z)| \quad \text{a.e. on } |z| = 1. \tag{2}$$

Since $f(z)$ is not identically 0 it follows that $F(z)$ is nonzero a.e. on $|z|=1$. Thus we may divide (2) by $F(z)$ and obtain

$$B(z)F(z) = \overline{F(z)} \quad \text{a.e. on } |z| = 1. \tag{3}$$

But the left side of (3) is H^1 and so has all negative Fourier coefficients 0, the right side is conjugate H^1 and so has all positive Fourier coefficients 0!.

Thus only the constant term remains and we conclude that both sides are constants. This is to say $B(z)F(z)$ and $F(z)$ are both constants and so indeed $f(z)=(B(z)F(z)).\ F(z)$ is a constant.

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