and hence is a subvariety. If $Z = \bigcup_q Z^q$ is the decomposition into pure dimensional subvarieties, where $\text{codim } Z^q = q$, then since at points not in $\sigma(Z)$, Z is pure dimensional, $\sigma(Z) = (\bigcup_q \sigma(Z^q)) \cup (\bigcup_{q \neq r} (Z^q \cap Z^r))$. Q.E.D.

REFERENCE

1. R. C. Gunning and H. Rossi, *Analytic functions of several complex variables* Prentice-Hall, Englewood Cliffs, N. J., 1965.

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ON THE MAGNITUDE OF $x^n - 1$ IN A NORMED ALGEBRA

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In their work on topological near-rings, Beidleman and Cox¹ had occasion to prove the following theorem: If A is a linear transformation on C^n (with a norm $\|\cdot\|$) and if $\|A^n - I\| \leq \alpha$ for all n with $0 \leq \alpha < 1$, then $A = I$. Their proof consists in noting that $A - I$ is nilpotent and then examining the Jordan form.

It is certainly natural to inquire whether this theorem extends to the case where A is a bounded linear transformation on a normed linear space, or is in a normed algebra. Below we give an extremely short proof that the theorem does extend.

THEOREM. *If X is a normed algebra with 1 and $\alpha_n = \|x^n - 1\|$ satisfies $\alpha_n = o(n)$ and $\liminf n^{-1}(\alpha_1 + \alpha_2 + \cdots + \alpha_{n-1}) = \beta < 1$, then $x = 1$. In particular $x = 1$ if $\alpha_n \leq \alpha < 1$.*

PROOF. Write

$$\begin{aligned} (x - 1) &= \frac{x^n - 1}{n} + (x - 1) \left[1 - \frac{1 + x + \cdots + x^{n-1}}{n} \right] \\ &= \frac{(x^n - 1)}{n} + \frac{(x - 1)}{n} [(1 - x) + (1 - x^2) + \cdots + (1 - x^{n-1})]. \end{aligned}$$

Hence $\|x - 1\| \leq [\alpha_n + \|x - 1\|(\alpha_1 + \alpha_2 + \cdots + \alpha_{n-1})]/n$. Letting n tend to ∞ through a suitable sequence, we get $\|x - 1\| \leq \beta \|x - 1\|$ giving the result.

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¹ Private communication.