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ON COHOMOLOGICAL TRIVIALITY

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Let G be a finite group and A be a G -module. It is well known that if for some two consecutive dimensions the cohomology groups $H^r(H, A)$ are trivial for all subgroups H of G , then $H^r(H, A)$ are trivial for all dimensions r and for all subgroups H of G . This note is to point out the following generalization.

THEOREM. *Let G be a finite group and A be a G -module. If for some integer k and for some odd positive integer d , $H^k(H, A)$ and $H^{k+d}(H, A)$ are trivial for all subgroups H of G , then $H^r(H, A)$ are trivial for all integers r and for all subgroups H of G .*

PROOF. As usual we proceed by induction on the order $n = |G|$. The theorem is trivial for $n = 1$. Suppose $n > 1$ and assume the truth of the theorem for all groups of order $< n$. In particular, we may assume that $H^r(H, A) = 0$ for every dimension r and for every proper subgroup H of G . If n is not a prime power, then every Sylow subgroup of G is a proper subgroup and the conclusion follows by a well-known result.

Suppose now n is a prime power, so that G is solvable. Let H be a proper normal subgroup of G such that the quotient group G/H is cyclic. Since $H^r(H, A) = 0$ for every r , we have the Fundamental Exact Sequence of Hochschild and Serre [2];

$$0 \rightarrow H^r(G/H, A^H) \rightarrow H^r(G, A) \rightarrow H^r(H, A)$$

for every $r > 0$. Since the last term is trivial, this says that $H^r(G/H, A^H) \cong H^r(G, A)$ for every $r > 0$. By dimension shifting we could have assumed that $k > 0$. Thus we have that $H^k(G/H, A^H) = H^{k+d}(G/H, A^H) = 0$. Since G/H is cyclic and d is odd, we obtain

that $H^r(G/H, A^H) = 0$ for every r . But then by the isomorphism $H^r(G, A) = 0$ for every $r > 0$. Once we have this, then we obtain that $H^r(G, A) = 0$ for every $r \leq 0$ as well. Q.E.D.

The theorem suggests the following conjecture.

CONJECTURE. Let G be a finite group. Let A and B be G -modules and let $f: A \rightarrow B$ be a G -homomorphism. If f induces isomorphisms

$$H^r(H, A) \cong H^r(H, B)$$

for some two dimensions that differ by an odd integer and for all subgroups H of G , then f induces isomorphisms for all dimensions r and for all subgroups H of G .

The theorem proved is the special case of the conjecture when B is trivial. L. Evens [1] proved the conjecture when the difference of the dimensions is 1. The conjecture is easily seen to be true when G has a cohomological period by reducing it to the case when G is a p -group.

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