NOTE ON THE ANNULUS CONJECTURE¹

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We give here a partial solution of the annulus conjecture which includes all known partial solutions except those assuming a condition equivalent to the annulus conjecture.

Lipschitz (or $C^{1/2}$) *manifolds* and *maps* are defined in [4]. By a *Lipschitz submanifold* of a Euclidean space we mean a C^r submanifold in the sense of [3] with $r = \frac{1}{2}$. A 1-1 Lipschitz mapping of one metric space onto another is said to be *regular* if and only if its inverse is Lipschitz.

THEOREM. *An n -dimensional annular region in R^n is an annulus if each of its bounding spheres contains a non-empty open subset which is a Lipschitz submanifold of R^n .*

PROOF. Let S be a bounding sphere of our annular region. We identify $S \times (-1, 1)$ with the standard open n -annulus $S^{n-1} \times (-1, 1)$ by a homeomorphism $(x, t) \rightarrow (f(x), t)$ where $x \in S$, $t \in (-1, 1)$, and $f: S \rightarrow S^{n-1}$ is a homeomorphism. It follows from our hypothesis that there is a regular Lipschitz homeomorphism $g: U \times (-1, 1) \rightarrow R^n$ such that $g(x, 0) = x$ for all $x \in U$. Let $V \subset U$ be a nonempty open subset of S such that $\bar{V} \subset U$. Then, by Brown [1], there is a mapping $h: S \times (-1, 1) \rightarrow R^n$ such that $h(x, 0) = x$ for all $x \in S$ and $h(x', t) = g(x', t)$ for all $x' \in V$, $t \in (-1, 1)$. By Rademacher's theorem [5, p. 272] and the chain rule, the mapping $h|V \times (-1, 1)$ is differentiable with nonvanishing Jacobian almost everywhere. By Kirby [2], our annular region is an annulus.

REFERENCES

1. M. Brown, *Locally flat embeddings of topological manifolds*, Ann. of Math. **75** (1962), 331-341.
2. R. C. Kirby, *On the annulus conjecture*, Proc. Amer. Math. Soc. **17** (1966), 178-185.
3. J. Munkres, *Elementary differential topology*, Princeton Univ. Press, Princeton, N. J., 1963.
4. J. H. C. Whitehead, *Manifolds with transverse fields in Euclidean space*, Ann. of Math. **73** (1961), 154-212.
5. H. Whitney, *Geometric integration theory*, Princeton Univ. Press, Princeton, N. J., 1957.

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