ON THE HYPERSPACE OF SUBCONTINUA OF THE PSEUDOARC

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Andrew Conner [1, p. 152] has raised the question "can the hyperspace of subcontinua of the pseudoarc (with the Hausdorff metric) be embedded in E^3 ?" This question is answered in the affirmative. The proof depends on a result of Hamstrom and Dyer [2, Theorem 8, p. 116] which readily implies that if G is an upper-semicontinuous decomposition of E^3 such that each element of G is a continuum which is contained in a horizontal plane and does not separate that plane then E^3/G is homeomorphic to E^3 . Using this we obtain the following theorem.

THEOREM. *Let P be a hereditarily indecomposable subcontinuum of the plane which does not separate the plane. Let $C(P)$ denote the space of subcontinua of P with the Hausdorff metric. Then $C(P)$ can be topologically embedded in E^3 .*

We first prove the following lemma.

LEMMA. *Let M be a metric continuum and p a point of M . Then the set of subcontinua of M which contain p is connected under the Hausdorff metric.*

PROOF. Suppose this set is written as the union of two mutually separated sets H and K . We may suppose that $M \in H$. Let T be a maximal element of K (i.e. an element of K which is contained in no other element of K ; such exist since K is compact). Then T would have to be a limit point of H since the closure of the component containing T of any neighborhood (in M) of T would properly contain T and hence be in H . This is a contradiction.

PROOF OF THEOREM. Following Whitney [3, p. 275] let μ be a function from $C(P)$ to the nonnegative reals such that $\mu\{p\} = 0$ if $p \in P$, $\mu(P) = 1$, μ is continuous, and if $A \subseteq B$ then $\mu(A) \leq \mu(B)$. It follows from the Lemma that if $p \in P$ and $0 \leq \epsilon \leq 1$ then there is an element h of $C(P)$ such that $p \in h$ and $\mu(h) = \epsilon$. Since P is hereditarily indecomposable there is exactly one such h , for if h and h' are elements of $C(P)$ which meet then $h \subset h'$ or $h' \subset h$ and if $\mu(h) = \mu(h')$ then $h = h'$. Let $Q (= P \times I)$ be the set of all points (x, y, z) in E^3 such that $(x, y) \in P$ and $0 \leq z \leq 1$. Let the upper-semicontinuous decomposition

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G of E^3 be defined by $G = G_1 \cup G_2$ where G_1 is the set of all points not in Q and G_2 is the set of all continua of the form $g \times \mu(g)$ where $g \in C(P)$. The above remarks show that each point of Q is in precisely one element of G_2 . To see that G_2 is actually a continuous decomposition of Q suppose that $g_1, g_2, \dots$ is a sequence of elements of G_2 which converges to the set h . For each i , $g_i = g_i' \times \mu(g_i')$ where $g_i' \in C(P)$. Now $g_1', g_2', \dots$ converges to an element h' of $C(P)$ and $\lim_{i \rightarrow \infty} \mu(g_i') = \mu(\lim_{i \rightarrow \infty} g_i') = \mu(h')$ so $h = h' \times \mu(h')$ and thus $h \in G_2$. Thus G is an upper-semicontinuous decomposition of E^3 and each element of G is contained in a horizontal plane and does not separate that plane and thus there is a homeomorphism f from E^3/G onto E^3 . Let $\phi: C(P) \rightarrow E^3/G$ be defined by $\phi(g) = g \times \mu(g)$. Then ϕ is 1-1 and continuous, so if $T: C(P) \rightarrow E^3$ is defined by $T(g) = f(\phi(g))$, T is a homeomorphism.

REFERENCES

1. R. H. Bing and R. J. Bean, Editors, *Topology Seminar, Wisconsin*, 1965, Annals of Mathematics Studies No. 60, Princeton Univ. Press, Princeton, N. J., 1966.
2. E. Dyer and M.-E. Hamstrom, *Completely regular mappings*, Fund. Math. **45** (1958), 103-118.
3. H. Whitney, *Regular families of curves*, Proc. Nat. Acad. Sci. U.S.A. **18** (1932), 275-278.

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