ANALYTIC FUNCTIONS AND JORDAN ARCS

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Let J be a Jordan arc lying on the complex sphere S , and let $F: S \rightarrow S$ be a homeomorphism. In several different connections (see e.g., [4]) it has been asked if the conformality of F off J implies that F is a linear fractional transformation. The purpose of this brief note is to point out that a recent example of Rothberger [5] can be used to answer this question in the negative.

Rothberger proves the

THEOREM. *There exists a function $F: S \rightarrow S$ such that*

- (1) *F is an autohomeomorphism of S ;*
- (2) *F is conformal off a totally disconnected perfect subset E of $S \setminus \{\infty\}$;*
- (3) *F is not a linear fractional transformation.*

By a theorem of Antoine [1, p. 302], E lies on a simple Jordan arc J . Taking this J and the F of the theorem, we obtain the required example.

A second, less explicit construction also deserves mention. Suppose there exists a (nonconstant) continuous function f on S , bounded and analytic on $S \setminus J$, which satisfies a Lipschitz condition

$$|f(z) - f(w)| < M |z - w|, \quad z, w \in S \setminus \{\infty\},$$

M an absolute constant. Then it is easy to see that the function $F(z) = Mz + f(z)$ is a nontrivial autohomeomorphism of S which is conformal off J . The interesting question of necessary and sufficient conditions (on J) for the existence of such an f seems to be open; related problems have been studied by Carleson [2] and Dolzhenko [3].

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