SHORTER NOTES

The purpose of this department is to publish very short papers of an unusually elegant and polished character, for which there is no other outlet.

REDEI'S FINITENESS THEOREM FOR COMMUTATIVE SEMIGROUPS

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Redei proved in [1] that

THEOREM. *Every finitely generated commutative semigroup is finitely presented.*

The statement has a short proof.

Suppose it is false. There would exist a free commutative semigroup of rank n with an ascending chain of congruences, or equivalently, with a descending chain of quotient semigroups:

$$F_n \rightarrow S_1 \rightarrow S_2 \rightarrow \cdots$$

Consider the semigroup rings over the integers,

$$Z[F_n] \rightarrow Z[S_1] \rightarrow \cdots$$

We have a descending chain of quotient rings of $Z[F_n]$, hence an ascending chain of ideals. But $Z[F_n]$ is the polynomial ring on n variables. By Hilbert's Basis theorem the chain of ideals must terminate, and we are done.

BIBLIOGRAPHY

1. Laszlo Redei, *The theory of finitely generated commutative semigroups*, Oxford Univ. Press, New York, 1965.

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