A NOTE ON A THEOREM OF KOKORIS

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The purpose of this note is to give a simple proof of the following result of Kokoris, which is part of Theorem 5.6 of [1]. In what follows, all algebras are assumed to be finite-dimensional.

THEOREM. *If A is a simple nodal noncommutative Jordan algebra of the form $F1+N$ over a field F of characteristic $\neq 2$, then A^+ is associative.*

First we note that A^+ is D -simple, i.e. $A^+A^+ \neq 0$, and A^+ does not have any characteristic ideals other than the zero ideal and A^+ . Secondly, N^+ is the radical of A^+ [1, pp. 143–146]. Thus the theorem is proved if we prove the

LEMMA. *If A is a D -simple nonassociative algebra of the form $F1+R$ (1 is the identity of A and R is an ideal of A), then A is associative and commutative.*

PROOF. Firstly, if B is a subspace of A , then the ideal I generated by B in A is the subspace $B+BA+AB+A(BA)+A(AB)+(BA)A+(AB)A+\dots$; if B is, in addition, a characteristic subspace, then the ideal I is also characteristic. If B is contained in the ideal R then so is I . Thus, by D -simplicity, R contains no nonzero characteristic subspaces of A . But the subspace spanned by all associators (x, y, z) ($(x, y, z) = xy \cdot z - x \cdot yz$, x, y, z in A) and the subspace spanned by all commutators (x, y) ($(x, y) = xy - yx$) are both characteristic subspaces, and both are contained in R ; so they are both zero, and A is associative and commutative.

The ideal R of the lemma is, in fact, the radical of A . More generally, any proper ideal B of a D -simple associative algebra A is solvable. For, if $B^{(1)} = B, \dots, B^{(k)} = B^{(k-1)}B^{(k-1)}, \dots$ is the derived series of B , $B^{(k)}$ are ideals of A , and $B \supseteq B^{(2)} \supseteq \dots$ is a descending chain of ideals. A being finite-dimensional, $B^{(k)} = B^{(k+1)}$ for some k . If D is a derivation of A , $B^{(k)}D = B^{(k+1)}D = (B^{(k)}B^{(k)})D \subseteq B^{(k)}D \cdot B^{(k)} + B^{(k)} \cdot B^{(k)}D \subseteq B^{(k)}$, i.e. $B^{(k)}$ is a characteristic ideal of A . Since A is D -simple, $B^{(k)} = 0$. Now, the following characterisation is an easy consequence of the lemma and a result of Harper [1, p. 146]: *If A is a D -simple nonassociative algebra of the form $F1+R$ for an ideal R of A , then A is the commutative associative algebra $F[1, x_1, \dots, x_n]$, $x_i^2 = 0$ for some n .*

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More applications of the lemma and an alternative but equally simple proof of the theorem will be given in a subsequent paper of the author. The author is indebted to the referee for his valuable suggestions.

REFERENCE

1. R. D. Schafer, *An introduction to nonassociative algebras*, Academic Press, New York, 1966.

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