A NOTE ON A PAPER OF PAUL HILL AND CHARLES MEGIBBEN

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This note deals with the correction of a small flaw in the proof of a theorem (and a lemma) in [2]. Also a simpler proof of the theorem in question is given. In the proof of Theorem 2.3 in [2], the statement "that distinct L 's cannot belong to a common pure subgroup H_L follows immediately from the fact that no two L 's generate with $G[p]$ the same subgroup of G " is false as it stands. One has to exercise a little more care in the selection of H_L . For example, since the union $[x_\alpha, x'_\alpha]_{\alpha \in A}$ of all the L 's (in Lemma 2.1 in [2]) is still linearly independent, there exists according to [1] a pure subgroup H of G supported by S such that $H \supseteq L$ for each L . Hence one could choose $H_L = H$ for each L . However, all goes well if H_L is chosen such that $H_L \cap [x_\alpha, x'_\alpha]_{\alpha \in A} = L$, as can easily be done. Similar remarks hold for Lemma 3.8 in [2].

A short alternate proof of Theorem 2.3 in [2] is given. In what follows let c be the cardinality of the continuum and Ω be the first ordinal whose cardinality is c . The notation and terminology will be the same as that in [2].

THEOREM (HILL AND MEGIBBEN). *Let G be a reduced primary group having a countable basic subgroup. If S is a proper dense subsocle of G such that $|S| = c$, then S supports 2^c isomorphically distinct pure subgroups of G .*

PROOF. Let H be a pure subgroup of G supported by S . Since G has a countable basic subgroup, $|pH \cap S| = c$. Thus

$$pH \cap S = \sum_{\alpha \in \Omega} \{x_\alpha\}.$$

For each $\alpha \in \Omega$, let $y_\alpha \in H$ such that $py_\alpha = x_\alpha$. Let $w \in G[p] - S$, and let $y'_\alpha = y_\alpha + w$. Let

$$\mathfrak{s} = \{[z_\alpha \mid \alpha \in \Omega] \mid z_\alpha \in [y_\alpha, y'_\alpha]\}.$$

For each $L \in \mathfrak{s}$, let H_L be a pure subgroup of G supported by S and containing $\{L\}$. Then $[H_L]_{L \in \mathfrak{s}}$ is a set of 2^c distinct subgroups of G . This is clear since if $L, L' \in \mathfrak{s}$ such that $L \neq L'$, then

$$\{L, L'\}[p] \not\subseteq S.$$

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The theorem then follows from the fact that $|\text{Hom}(H, G)| = c$ for any pure subgroup of G supported by S .

REFERENCES

1. Paul Hill and Charles Megibben, *Minimal pure subgroups in primary groups*, Bull. Soc. Math. France 92 (1963), 251–257.
2. ———, *On primary groups with countable basic subgroups*, Trans. Amer. Math. Soc. 124 (1966), 49–59.

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