ANOTHER SUMMABLE C_Ω -GROUP

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ABSTRACT. An example is given of a p -primary Abelian group G having the following properties: G is summable; the length of G is Ω ; the α th Ulm invariant of G is one for all $\alpha < \Omega$; if $\alpha < \Omega$, any $p^\alpha G$ -high subgroup of G is countable; $G/p^\alpha G$ is countable for all $\alpha < \Omega$; and G is not p^β -projective for any ordinal β .

In [4], Honda gives an existence theorem showing the existence of a summable p -primary Abelian group G of length Ω (the first uncountable ordinal) such that the α th Ulm invariant of G is one for all ordinals $\alpha < \Omega$. Thus we have an example of a p -primary Abelian group G with the following properties:

- (1) G is summable.
- (2) The length of G is Ω .
- (3) The α th Ulm invariant of G is one for all $\alpha < \Omega$.
- (4) If $\alpha < \Omega$, any $p^\alpha G$ -high subgroup of G is countable.
- (5) $G/p^\alpha G$ is countable for all $\alpha < \Omega$.
- (6) G is not p^β -projective for any ordinal β .

Note that (4) follows from (1) and (3); (5) follows from (4) and Corollary 1.5 of [1]; and (6) follows from Proposition 6.7 of [6].

In [5], Megibben developed a " C_λ theory" (for λ a countable ordinal) for torsion Abelian groups. A p -primary Abelian group G belongs to the class C_λ (λ any ordinal) if and only if $G/p^\alpha G$ is a direct sum of countable groups (d.s.c.) for all $\alpha < \lambda$. In the case λ countable, it was shown [3] that: if G is a summable C_λ -group of length λ then G is a d.s.c., and if G is summable of length λ and for each $\alpha < \lambda$, G contains a p^α -high subgroup which is a d.s.c., then G is a d.s.c. In [2], Hill has given an example to show that the countability condition on λ cannot be removed.

The above example is a very striking example of a p -primary Abelian group G of length Ω satisfying Megibben's criterion that is not a d.s.c. (see [7]). In fact (4) and (5) are stronger conditions than posed by Megibben's criterion; and G is not only not a d.s.c., it is not p^β -projective for any ordinal β .

Received by the editors January 15, 1970.

AMS 1969 subject classifications. Primary 2030.

Key words and phrases. Summable, p -primary Abelian group, length Ω , not p^β -projective.

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