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ADDENDUM TO "ON THE FRATTINI SUBGROUP"

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ABSTRACT. Let F be a free group, R a normal subgroup of F and V a fully invariant subgroup of R. In a recent paper the authors calculated the Frattini subgroup of F/V under suitable conditions on R and V. This paper presents information on the Frattini subgroup of subgroups of F/V under the same conditions.

1. **Introduction.** The main result is (cf. [2, Theorem 1]: we also follow the notation of [2]).

THEOREM 1. *Let F be a noncyclic free group, R a normal subgroup of F such that F/R is residually finite. If S/R' is a subgroup of F/R', then the Frattini subgroup of S/R' is trivial.*

Armed with this result, we can generalise the remaining results of [2]: as a sample, we state

THEOREM 2. *Let $V_1, \cdots, V_n$ be nilpotent varieties, with V_1 of exponent* 0, *and let F be a free group. Put $V=V_1 \cdots V_n$, $R=V_2 \cdots V_n(F)$. Then $\Phi(S/V(F)) \leqq R'/V(F)$: in particular, any subgroup of a relatively free group of V has nilpotent Frattini subgroup.*

COROLLARY 3 (SOKOLOV [4, THEOREM 3]). *Subgroups of free soluble groups have trivial Frattini subgroup.*

The proofs of Theorem 2 and Corollary 3 are easily modified from the proofs of the corresponding results in [2].

2. **The proof of Theorem 1.** The proof separates into two cases:
(1) $S\cap R=R'$.
(2) $S\cap R\neq R'$.
For the first case, we have that the sequence

$$1 \to R/R' \to SR/R' \to S/R' \to 1$$

is exact, and also splits. But Graham Higman [3] has shown that S/R' then has cohomological dimension 1, and so S/R' is free (Stallings [5] and Swan [6]). It is well known that the Frattini subgroup of a free group is trivial, and so we are left with case (2).

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Let C/R' denote the centraliser in S/R' of $S\cap R/R'$. Then if $1\neq g\in S\cap R/R'$, or $g\notin C/R'$, the proof of Theorem 1 of [2] may be applied to show that $g\notin\Phi(S/R')$. Thus we need to show that if $g\in C/R'\backslash R/R'$, then $g\notin\Phi(S/R')$. For such a g, consider $\langle g, S\cap R/R'\rangle =H/R'$. Theorem 2 of Baumslag and Gruenberg [1] gives us that $H/S\cap R$ is a finite cyclic group, and so $g^n\in(S\cap R)/R'$ for some integer $n>1$. But we know that F/R' is torsion free (Graham Higman [3]), and hence $g^n\neq 1$. If $g\in\Phi(S/R')$ then $g^n\in\Phi(S/R')\cap(S\cap R/R') =1$, a contradiction.

This completes the proof of Theorem 1.

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