REGULAR MATRICES WITH NOWHERE DENSE SUPPORT

JOHN RAINWATER

ABSTRACT. A regular matrix has nowhere dense support in $\beta N \setminus N$ if and only if the largest entry in the m th row converges to 0.

R. E. Atalla has asked [1] whether for every regular matrix $A = (a_{mn})$ whose rows spread (meaning $\lim_m \max_n |a_{mn}| = 0$), the support set is nowhere dense in $\beta N \setminus N$. This means that every infinite subset S of N has an infinite subset T such that for every bounded sequence $x: N \rightarrow R$ vanishing outside T the transform Ax (where $Ax(m) = \sum_n a_{mn}x(n)$) is a null sequence.

PROOF. Since $b_m = \max_n |a_{mn}| \rightarrow 0$, there are indices $m_1 < m_2 < \dots$ such that for $m > m_k$, $b_m < 2^{-2k}$. Since A is regular, for each k , uniformly for $i = m_k + 1, \dots, m_{k+1}$, there is j_k such that $\sum_{j=j_k}^{\infty} |a_{ij}| < 2^{-k}$. Any infinite set S contains an increasing sequence (t_k) with $t_k \geq j_k$. If x is supported by $\{t_k\}$ and has sup norm M , then $|Ax(m)| < 2^{1-k}M$ for $m > m_k$; for in $n < j_k$, $x(n)$ is nonzero at most $k-1$ times and $|a_{mn}x(n)|$ adds up to less than $2^{-k}M$, and for $n \geq j_k$ the sum is no more than $2^{-k}M$.

Note that if the rows do not spread there are entries a_{mn} with $m \rightarrow \infty$ such that $|a_{mn}| > \epsilon > 0$. By regularity, $n \rightarrow \infty$ also, and the derived set of the n 's is interior to the support set.

I am indebted to Dr. Atalla for some correspondence motivating the problem.

REFERENCE

1. R. E. Atalla, *On the multiplicative behavior of regular matrices*, Proc. Amer. Math. Soc. **26** (1970), 437-445.

UNIVERSITY OF WASHINGTON, SEATTLE, WASHINGTON 98105

Received by the editors May 25, 1970.

AMS 1970 subject classifications. Primary 40C05.

Key words and phrases. Infinite matrix, support set, βN , nowhere dense.