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ANOTHER FIXED POINT THEOREM FOR PLANE CONTINUA

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ABSTRACT. A continuum M is said to be λ connected if every two points of M can be joined by a hereditarily decomposable subcontinuum of M. Here we prove that a bounded plane continuum that does not have infinitely many complementary domains is λ connected if and only if its boundary does not contain an indecomposable continuum. It follows that every λ connected bounded nonseparating subcontinuum of the plane has the fixed point property.

If a nondegenerate point set is both connected and closed it is called a continuum. A set X is said to have the fixed point property if for each map $f: X \to X$ there is a point $x \in X$ such that $f(x) = x$. H. Bell proved [**1**] that every bounded nonseparating plane continuum that has a hereditarily decomposable boundary has the fixed point property (for a different proof see [**3**]). Recently the author proved [**2**] that every bounded nonseparating plane continuum that is arcwise connected has a hereditarily decomposable boundary and therefore has the fixed point property. In this note the author's theorem is extended to λ connected bounded nonseparating plane continua.

THEOREM 1. *Suppose M is a bounded continuum in the plane S that does not have infinitely many complementary domains. Then M is λ connected if and only if* Bd M *(the boundary of M) does not contain an indecomposable continuum.*

PROOF. Suppose Bd M does not contain an indecomposable continuum. Then Bd M is the union of a finite number of hereditarily decomposable continua. Let B_1 denote a component of Bd M; B_1 is hereditarily decomposable. Let A_1 be an arc in S irreducible from B_1 to Bd $M - B_1$. Since only the endpoints of A_1 belong to Bd M and they lie in the boundaries of different complementary domains of M, the arc A_1 is a subset of M. Continuing this process it is clear that the union of a finite number of arcs

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with Bd M produces a hereditarily decomposable subcontinuum H of M. Clearly any interior point of M may be joined to H by an arc in M. Hence M is λ connected.

Assume that M is λ connected. Suppose there exists an indecomposable continuum I in Bd M. Let q be a point of $M-I$. Let $\{U_n\}$ be the elements of a countable base for the topology on S that intersect I. Since M is λ connected, for each point p of I, there exists a subcontinuum L of M that contains $\{p, q\}$ and does not contain I. For each positive integer n, let H_n be the set of all points of I that can be joined with q by a continuum in $M-U_n$. Note that $I=\bigcup_{n=1}^{\infty} H_n$. For some integer j, the closure of H_j contains a nonempty open subset of I. Hence there exists a continuum in $M-U_j$ that contains a nonempty open subset of I. Since every subcontinuum of M that contains a nonempty open subset of I contains I [**2**, Theorem 1], this is a contradiction. Therefore Bd M does not contain an indecomposable continuum.

THEOREM 2. *Every bounded λ connected nonseparating plane continuum has the fixed point property.*

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