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THE HYPERSPACE OF A PSEUDOARC IS A CANTOR MANIFOLD[1]

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ABSTRACT. The following theorem which was conjectured by C. Eberhart and S. B. Nadler, Jr., in [EN] is proved.

THEOREM. *The hyperspace of nonvoid subcontinua of a pseudoarc is a two-dimensional Cantor manifold.*

1. **Introduction.** The hyperspace $C(X)$ of nonvoid subcontinua of a metric continuum X has been investigated extensively. (We will restrict our discussion to metric continua.) It is known that $C(X)$ is always compact and arcwise connected [**KE**]. The basic work [**S**] establishes the relationship between $C(X)$ and inverse limit spaces. Inverse limit methods have yielded further properties of $C(X)$. Namely, $C(X)$ is acyclic in all dimensions [**S**], unicoherent [**S**], [**N**], and has dimension exceeding one for nondegenerate X [**EN**]. By specializing X, much more can be said of $C(X)$. Notable works along this line are [**D1**] and [**D2**] where X is locally connected. The hyperspace of an hereditarily indecomposable continuum X also has been studied. See [**KE**], [**EN**], [**R**], [**T**] and [**H**]. The present paper concerns itself with one such hereditarily indecomposable continuum, the pseudoarc. It is known that the hyperspace of a pseudoarc is embeddable in Euclidean three-dimensional space [**T**], [**H**] and that its dimension is two [**EN**]. We add to the large collection of facts about $C(X)$ the theorem stated in the abstract. This theorem improves the dimension two assertion of [**EN**].

2. **The function μ.** Let X be a nondegenerate metric continuum and $C(X)$ be the space of all nonvoid subcontinua of X with the Hausdorff metric [**KU**]. In [**KE**], Kelley noted the existence (originally due to Whitney [**W**]) of a real-valued function μ defined on $C(X)$ and having

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the following properties:

(1) μ is continuous;

(2) if A, $B\in C(X)$, $A\subset B$ and $A\neq B$, then $\mu(A)<\mu(B)$;

(3) $\mu(X)=1$;

(4) $\mu(\{x\})=0$ for each $x\in X$.

He proved among others that

(a) $C(X)$ is an arcwise connected continuum;

(b) if X is hereditarily indecomposable, A, $B\in C(X)$, $A\cap B\neq\varnothing$ and $\mu(A)=\mu(B)$ then $A=B$;

(c) X is hereditarily indecomposable if and only if $C(X)$ contains a unique arc between every pair of its elements.

Suppose further that X is a pseudoarc. Then, in **[R]**, it was observed that the space $\mu^{-1}(t)$, $0\leqq t<1$, is a totally pathwise disconnected continuum. Subsequently, Eberhart and Nadler in **[EN]** observed that $\mu^{-1}(t)$ is a continuous decomposition of X and hence, by **[B1]**,

(d) $\mu^{-1}(t)$ is a pseudoarc for $0\leqq t<1$ whenever X is a pseudoarc.

We now prove three lemmas which will be needed later.

LEMMA 2.1. *Suppose X is a pseudoarc and $0\leqq t<1$. Then there is a homeomorphism h_t of $C(X)$ onto $\mu^{-1}[t,1]=\{A\in C(X):t\leqq\mu(A)\}$ such that $h_t[\mu^{-1}(0)]=\mu^{-1}(t)$.*

PROOF. Using (d) above, we let $h:X\to\mu^{-1}(t)$ be a homeomorphism. Define a mapping $\bar{h}$ on $C(X)$ onto the hyperspace $C(\mu^{-1}(t))$ of the space $\mu^{-1}(t)$ by $\bar{h}(A)=h(A)$ for each $A\in C(X)$. Then $\bar{h}$ is a homeomorphism.

Let 2^X be the space of all nonvoid closed subsets of X with the Hausdorff metric, and 2^{2^X} be the space of all nonvoid closed subsets of 2^X with Hausdorff metric. Let $\sigma:2^{2^X}\to 2^X$ be defined by $\sigma(\mathscr{A})=\bigcup\{A\in 2^X:A\in\mathscr{A}\}$, $\mathscr{A}\in 2^{2^X}$. In **[KE]**, it is shown that σ is continuous. Since $C(\mu^{-1}(t))\subset 2^{2^X}$, we let σ be the restriction on $C(\mu^{-1}(t))$. Let $\mathscr{A}\in C(\mu^{-1}(t))$. Then $\sigma(\mathscr{A})$ is a subcontinuum of X, and thus $\sigma(\mathscr{A})\in C(X)$. Let $A\in\mathscr{A}$. Then $t=\mu(A)$ and $A\subset\sigma(\mathscr{A})$, so that by the property (2) of μ, $\mu(A)\leqq\mu(\sigma(\mathscr{A}))$. This implies that $\sigma(\mathscr{A})\in\mu^{-1}[t,1]$. If $A\in\mu^{-1}[t,1]$, then $\mu(A)=s$, $s\geqq t$. Let $\mathscr{A}=\{B\in\mu^{-1}(t):B\subset A\}$. For each $x\in A$, since the unique arc $\mathscr{A}_x$ in $C(X)$ from $\{x\}$ to X must meet $\mu^{-1}(t)$, there is an element $B\in\mu^{-1}(t)$ such that $x\in B$ and $B\subset A$ **[KE]**. We would like to show that $\mathscr{A}\in C(\mu^{-1}(t))$. Consider the projection mapping f of X onto the space $\mu^{-1}(t)$ defined by $f(x)=B$ if $x\in B$. This function is continuous **[R]**, and $f(A)=\mathscr{A}$. Since A is a subcontinuum of X, so is $\mathscr{A}$ in $\mu^{-1}(t)$. Therefore $\mathscr{A}\in C(\mu^{-1}(t))$. Thus $\sigma(\mathscr{A})=A$ and σ is a continuous mapping of $C(\mu^{-1}(t))$ onto $\mu^{-1}[t,1]$. The fact that σ is one-to-one follows from (b) above. Therefore $\sigma:C(\mu^{-1}(t))\to\mu^{-1}[t,1]$ is a homeomorphism.

We let $h_t: C(X) \to \mu^{-1}[t, 1]$ be the homeomorphism defined by $h_t = \sigma \circ \bar{h}$. The lemma is now proved.

LEMMA 2.2. *Suppose X is a pseudoarc and $0 \leqq t < 1$. Then there is a mapping $g_t: C(X) \to C(X)$ such that g_t restricted to $\mu^{-1}[t, 1]$ is h_t^{-1} and $g_t[\mu^{-1}[0, t]] = \mu^{-1}(0)$.*

PROOF. For each $A \in \mu^{-1}[0, t]$ there is a unique set $B_A \in \mu^{-1}(t)$ such that $A \subset B_A$. Let $g_t(A) = h_t^{-1}(B_A)$. It is clear that g_t is continuous on $\mu^{-1}[0, t]$. If g_t is defined to be h_t^{-1} on $\mu^{-1}[t, 1]$ then the desired mapping is constructed.

Since μ is a closed continuous mapping, we have immediately the following lemma.

LEMMA 2.3. *For each closed set F and open set $\mathcal{O} \supset \mu^{-1}[F]$, there is an open set Q such that $\mu^{-1}[F] \subset \mu^{-1}[Q] \subset \mathcal{O}$.*

Finally, we remark that, if $h: C(X) \to C(X)$ is a homeomorphism and X is a pseudoarc then necessarily $h[\mu^{-1}(0)] = \mu^{-1}(0)$ and $h(X) = X$.

3. **The dimension of the hyperspace of a pseudoarc.** In this section we prove two theorems concerning the dimension of the hyperspace $C(X)$ of a pseudoarc X. The first theorem has been established by Eberhart and Nadler [**EN**]. The present proof is new and relies only on properties of the pseudoarc. In the above-mentioned paper, it is observed that $C(X)$ is of dimension two at each point of $\mu^{-1}(0, 1) = \{A \in C(X): 0 < \mu(A) < 1\}$. The second theorem of this section shows that $C(X)$ is also of dimension two at X. This fact will be used in the next section to prove the main theorem.

THEOREM 3.1. *If X is a pseudoarc then the dimension of $C(X)$ is two.*

PROOF. Since $C(X)$ is a nondegenerate continuum, we have $\dim C(X) \geqq 1$. From Theorem VI.7 of [**HW**], we have

$$\dim C(X) \leqq \dim \mu[C(X)] + \sup \{\dim \mu^{-1}(t): 0 \leqq t \leqq 1\}.$$

From §2 above, we have that the right side of the above inequality is two since the dimension of a pseudoarc is one. We need to prove $\dim C(X) \neq 1$.

Suppose $\dim C(X) = 1$. Since $C(X)$ is contractible [**R**], any mapping on a subcontinuum of $C(X)$ into S^1 is inessential, and therefore each subcontinuum of $C(X)$ has property (b). Thus, each subcontinuum of $C(X)$ is unicoherent [**WH**, p. 226]. But there are subcontinua of $C(X)$ which are not unicoherent. For example, $\mu^{-1}(0) \cup \mathcal{A}_{x,y}$, where $\mathcal{A}_{x,y}$ is the unique arc in $C(X)$ between $\{x\}$ and $\{y\}$, $x \neq y$, $x, y \in X$. Since a pseudoarc contains no arc, $\mu^{-1}(0) \cap \mathcal{A}_{x,y} = \{x, y\}$. Consequently, $\dim C(X) \neq 1$ and the theorem is proved.

THEOREM 3.2. *If X is a pseudoarc then $C(X)$ has dimension two at the point X.*

PROOF. The proof is by contradiction. Since $C(X)$ is a nondegenerate continuum, we have that the dimension of $C(X)$ at X is no less than one. We will show that the assumption that $C(X)$ is of dimension one at the point X implies dim $C(X)=1$. The proof will be made in three parts.

Part 1. Let $0<t<1$. If $C(X)$ has dimension one at X then there are two disjoint open sets $\mathscr{S}$ and $\mathscr{T}$ of $C(X)$ such that $\mu^{-1}(0)\subset\mathscr{S}$, $\mu^{-1}[t, 1]\subset\mathscr{T}$, and their boundaries have dim $\mathrm{Bd}(\mathscr{S})=0=\dim \mathrm{Bd}(\mathscr{T})$.

PROOF. Let $\mathscr{O}$ be an open neighborhood of X such that $\mathscr{O}\subset\mu^{-1}[\frac{1}{2}, 1]$ and dim $\mathrm{Bd}(\mathscr{O})=0$. Then, if $\mathscr{P}$ is the complement of the closure of $\mathscr{O}$, $\mathscr{P}$ is an open set containing $\mu^{-1}(0)$ and dim $\mathrm{Bd}(\mathscr{P})=0$.

Let $P_0\in\mathscr{O}$ and $P_0\neq X$. As Eberhart and Nadler in [**EN**] observed, Theorem 15 of [**B1**] implies for each $A\in\mu^{-1}[t, 1]$, $A\neq X$, there exists a homeomorphism $h_A:C(X)\to C(X)$ such that $h_A(P_0)=A$. Associated with this homeomorphism are two disjoint open sets $\mathscr{O}_A=h_A(\mathscr{O})$ and $\mathscr{P}_A=h_A(\mathscr{P})$ for which $A\in\mathscr{O}_A$, $\mu^{-1}(0)\subset\mathscr{P}_A$, and dim $\mathrm{Bd}(\mathscr{O}_A)=0=\dim \mathrm{Bd}(\mathscr{P}_A)$. Now, $\{\mathscr{O}_A : A\in\mu^{-1}[t, 1], A\neq X\}$ is an open cover of the compact set $\mu^{-1}[t, 1]$. Let $\mathscr{O}_{A_1}, \cdots, \mathscr{O}_{A_n}$ be a subcover, $\mathscr{T}=\bigcup_{i=1}^n \mathscr{O}_{A_i}$ and $\mathscr{S}=\bigcap_{i=1}^n \mathscr{P}_{A_i}$. Then $\mathscr{S}$ and $\mathscr{T}$ are disjoint open sets with $\mu^{-1}(0)\subset\mathscr{S}$ and $\mu^{-1}[t, 1]\subset\mathscr{T}$. Since $\mathrm{Bd}(\mathscr{S})\subset\bigcup_{i=1}^n \mathrm{Bd}(\mathscr{P}_{A_i})$ and $\mathrm{Bd}(\mathscr{T})\subset\bigcup_{i=1}^n \mathrm{Bd}(\mathscr{O}_{A_i})$, we have dim $\mathrm{Bd}(\mathscr{S})=0=\dim \mathrm{Bd}(\mathscr{T})$ and the first part is proved.

Part 2. Let $0\leqq t\leqq 1$ and $\mathscr{O}$ be an open neighborhood of $\mu^{-1}(t)$. Suppose the conclusion of Part 1 holds. Then there is an open neighborhood $\mathscr{W}$ of $\mu^{-1}(t)$ such that $\mathscr{W}\subset\mathscr{O}$ and dim $\mathrm{Bd}(\mathscr{W})=0$.

PROOF. By Lemma 2.3, there are two numbers s_1 and s_2 such that $s_1<t<s_2$ and $\mu^{-1}[s_1, s_2]\subset\mathscr{O}$. We assume for convenience that $0<t<1$. The contrary cases involve only a slight modification of the argument. We may now further assume $0\leqq s_1<t<s_2\leqq 1$.

Let us consider s_1. By Lemma 2.2 there is a mapping $g_{s_1}:C(X)\to C(X)$ such that g_{s_1} maps $\mu^{-1}[s_1, 1]$ homeomorphically onto $C(X)$ and $g_{s_1}[\mu^{-1}[0, s_1]]=\mu^{-1}(0)$. $g_{s_1}[\mu^{-1}[t, 1]]$ is a closed set disjoint with $\mu^{-1}(0)$. Hence by Lemma 2.3 there is a number T_1 with $0<T_1$ such that $\mu^{-1}[0, T_1]\cap g_{s_1}[\mu^{-1}[t, 1]]=\varnothing$. From Part 1 there is an open set $\mathscr{T}$ such that the closure of $\mathscr{T}$ does not meet $\mu^{-1}(0)$, $\mathscr{T}\supset\mu^{-1}[T_1, 1]$ and dim $\mathrm{Bd}(\mathscr{T})=0$. Thus, if $\mathscr{W}_1=g_{s_1}^{-1}(\mathscr{T})$ then $\mathscr{W}_1$ is open, $\mu^{-1}(t)\subset\mathscr{W}_1\subset\mu^{-1}[s_1, 1]$ and dim $\mathrm{Bd}(\mathscr{W}_1)=0$.

Next, consider t. By Lemma 2.2 there is a mapping $g_t:C(X)\to C(X)$ such that g_t maps $\mu^{-1}[t, 1]$ homeomorphically onto $C(X)$ and $g_t[\mu^{-1}[0, t]]=\mu^{-1}(0)$. $g_t[\mu^{-1}[s_2, 1]]$ is a closed set disjoint with $\mu^{-1}(0)$. Hence by Lemma 2.3 there is a number T_2 with $0<T_2$ such that

$\mu^{-1}[0, T_2] \cap g_t[\mu^{-1}[s_2, 1]] = \varnothing$. From Part 1, there is an open set $\mathscr{S}$ such that the closure of $\mathscr{S}$ does not meet $\mu^{-1}[T_2, 1]$, $\mathscr{S} \supset \mu^{-1}(0)$ and $\dim \mathrm{Bd}(\mathscr{S}) = 0$. Thus, if $\mathscr{W}_2 = g_t^{-1}(\mathscr{S})$ then $\mathscr{W}_2$ is open, $\mu^{-1}(t) \subset \mathscr{W}_2 \subset \mu^{-1}[0, s_2]$ and $\dim \mathrm{Bd}(\mathscr{W}_2) = 0$.

Let $\mathscr{W} = \mathscr{W}_1 \cap \mathscr{W}_2$. Then $\mathscr{W}$ is open, $\mu^{-1}(t) \subset \mathscr{W} \subset \mu^{-1}[s_1, s_2] \subset \mathscr{O}$ and $\dim \mathrm{Bd}(\mathscr{W}) \leqq \dim \mathrm{Bd}(\mathscr{W}_1) + \dim \mathrm{Bd}(\mathscr{W}_2) = 0$. Thus Part 2 is proved.

Part 3. If $C(X)$ has dimension one at X then $\dim C(X) = 1$.

PROOF. Let $\mathscr{K} = \{\mu^{-1}(t) : 0 \leqq t \leqq 1\}$. Then $\mathscr{K}$ is a family of closed subsets of $C(X)$. By Part 2, each neighborhood of $\mu^{-1}(t)$ contains a neighborhood whose boundary has dimension zero. Since $\dim \mu^{-1}(t) \leqq 1$ for each t, we have, by Proposition G on p. 90 of [**HW**], $\dim C(X) = \dim \bigcup \mathscr{K} \leqq 1$, a contradiction to Theorem 3.1. Thus Theorem 3.2 is proved.

4. **Proof of the main theorem.** We are now in a position to prove our main theorem. Lemma 2.3 provides us with the fact that the family $\mu^{-1}[t, 1]$, $0 \leqq t < 1$, forms a basis of closed neighborhoods of the point X in $C(X)$. We infer from Lemma 2.1 that we need only consider the neighborhood $C(X)$.

THEOREM 4.1. *If X is a pseudoarc then $C(X)$ is a two-dimensional Cantor manifold.*

PROOF. By denying the conclusion, we will establish a contradiction to Theorem 3.2. That is, we will show that the existence of a zero-dimensional separator of $C(X)$ implies the existence of an open neighborhood of X, disjoint with $\mu^{-1}(0)$, whose boundary has dimension zero. Then the preliminary remarks of this section will complete the proof.

Suppose $\mathscr{S}$ is a closed zero-dimensional subset of $C(X)$ which separates $C(X)$. Let $\mathscr{A}$ and $\mathscr{B}$ be nonvoid open sets such that $C(X) - \mathscr{S} = \mathscr{A} \cup \mathscr{B}$. We will consider two cases.

Case 1. Suppose $X \notin \mathscr{S}$. Without loss of generality, we may assume $X \in \mathscr{A}$. There are now two possibilities. Either $\mathscr{S} \cap \mu^{-1}(0) = \varnothing$ or $\mathscr{S} \cap \mu^{-1}(0) \neq \varnothing$. Let us dispose of the first possibility.

(a) *Suppose* $\mathscr{S} \cap \mu^{-1}(0) = \varnothing$. In the event that $\mu^{-1}(0) \subset \mathscr{B}$, the desired neighborhood of X is $\mathscr{A}$ and the contradiction is established. Since $\mu^{-1}(0)$ is connected, $\mu^{-1}(0) \not\subset \mathscr{B}$ implies $\mu^{-1}(0) \subset \mathscr{A}$. $\mathscr{B}$ being nonvoid, choose $P \in \mathscr{B}$. P is a nondegenerate subcontinuum of X since $P \notin \mu^{-1}(0)$. Hence P is a pseudoarc. $C(P)$ is homeomorphic to $C(X)$ and $C(P)$ is a subspace of $C(X)$. Clearly, $\mathscr{B} \cap C(P)$ is an open neighborhood of P in $C(P)$, disjoint with $C(P) \cap \mu^{-1}(0) = \{\{p\} : p \in P\}$, whose boundary in $C(P)$ has dimension zero. Hence, the required neighborhood of X exists and the contradiction is established. Thus, we have disposed of the possibility $\mathcal{S}\cap\mu^{-1}(0)=\varnothing$.

(b) *Suppose* $\mathcal{S}\cap\mu^{-1}(0)\neq\varnothing$. Either $\mathcal{B}\cap\mu^{-1}(0)\neq\varnothing$ or $\mathcal{B}\cap\mu^{-1}(0)=\varnothing$. Suppose first that $\mathcal{B}\cap\mu^{-1}(0)\neq\varnothing$. Let $P_0\in\mathcal{B}\cap\mu^{-1}(0)$ and $A\in\mu^{-1}(0)$. Then, both P_0 and A are singleton subsets of X. Since X is homogeneous, there is a homeomorphism $h_A:C(X)\to C(X)$ such that $h_A(P_0)=A$. Associated with each such homeomorphism are two disjoint open sets $\mathcal{O}_A=h_A(\mathcal{A})$ and $\mathcal{P}_A=h_A(\mathcal{B})$ with the properties $X\in\mathcal{O}_A$ and $\dim \operatorname{Bd}(\mathcal{O}_A)=0$. Since $\{\mathcal{P}_A:A\in\mu^{-1}(0)\}$ is an open cover of the compact set $\mu^{-1}(0)$, there is a finite cover $\mathcal{P}_{A_1},\cdots,\mathcal{P}_{A_n}$. Let $\mathcal{O}=\bigcap_{i=1}^n\mathcal{O}_{A_i}$ and $\mathcal{P}=\bigcup_{i=1}^n\mathcal{P}_{A_i}$. Then $\mathcal{O}$ and $\mathcal{P}$ are disjoint open sets, $X\in\mathcal{O}$, $\mu^{-1}(0)\subset\mathcal{P}$ and $\dim \operatorname{Bd}(\mathcal{O})=0$. Thus, the desired neighborhood of X is found and the contradiction established. Next, suppose $\mathcal{B}\cap\mu^{-1}(0)=\varnothing$. Since $X\notin\mathcal{B}$ and $\mathcal{B}\neq\varnothing$, there is a nondegenerate subcontinuum $P\in\mathcal{B}$. P is a pseudoarc and $C(P)$ is a subspace of $C(X)$ which is homeomorphic to $C(X)$. Since $\dim[C(P)\cap\mu^{-1}(0)]=1$ and $\dim\mathcal{S}=0$, we have $[C(P)\cap\mu^{-1}(0)]-\mathcal{S}$ is a nonempty subset of $\mathcal{A}\cap C(P)$. By considering $C(P)$, $\mathcal{S}'=C(P)\cap\mathcal{S}$, $\mathcal{A}'=\mathcal{B}\cap C(P)$ and $\mathcal{B}'=\mathcal{A}\cap C(P)$, we see that $\mathcal{S}'$ is a zero-dimensional separator of $C(P)$, $C(P)-\mathcal{S}'=\mathcal{A}'\cup\mathcal{B}'$ where $\mathcal{A}'$ and $\mathcal{B}'$ are open sets, $P\in\mathcal{A}'$ and $\mathcal{B}'\cap\mu_P^{-1}(0)\neq\varnothing$ where μ_P is a μ function associated with the pseudoarc P. We have arrived at the situation which immediately preceded the one at hand.

Now the two possibilities (a) and (b) under Case 1 have been completely disposed of.

Case 2. Suppose $X\in\mathcal{S}$. We will dispose of this case by reducing it to Case 1.

For each $x\in X$, there is a unique arc $\mathcal{A}_x$ in $C(X)$ from $\{x\}$ to X [**KE**]. Let $M=\{x\in X:\mathcal{A}_x\cap\mathcal{A}\neq\varnothing\}$ and $N=\{x\in X:\mathcal{A}_x\cap\mathcal{B}\neq\varnothing\}$. Since $\dim\mathcal{S}=0$, we have $\varnothing\neq\mathcal{A}_x-\mathcal{S}\subset\mathcal{A}\cup\mathcal{B}$ for each $x\in X$. Consequently, $X=M\cup N$. We will show $M\neq\varnothing$ and open. A symmetric argument shows $N\neq\varnothing$ and open. To this end, we recall a continuous mapping $\Phi:X\times[0,1]\to C(X)$ defined in Theorem 3.5 of [**R**]. Φ is defined as

$$\Phi(x,t)=A,\quad \text{where } x\in A\in C(X) \text{ and } \mu(A)=t.$$

Since each pair of points in $C(X)$ has a unique arc between them, we have $\mathcal{A}_x=\Phi[\{x\}\times[0,1]]$. Consequently, $M=F[\Phi^{-1}(\mathcal{A})]$, where F is the natural projection $F:X\times[0,1]\to X$.

Since X is connected $M\cap N\neq\varnothing$. Let $x\in M\cap N$ and $P\in\mathcal{A}_x\cap\mathcal{A}$ and $Q\in\mathcal{A}_x\cap\mathcal{B}$. Since P and Q are in the arc $\mathcal{A}_x$, either $P\supset Q$ or $P\subset Q$. Also, $P\neq Q$. Suppose $P\supset Q$. By considering the pseudoarc P, we have for $C(P)$, $\mathcal{S}'=C(P)\cap\mathcal{S}$, $\mathcal{A}'=\mathcal{A}\cap C(P)$, $\mathcal{B}'=\mathcal{B}\cap C(P)$, precisely the Case 1. Similar considerations apply when $P\subset Q$.

The main theorem is now established.

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