SHORTER NOTES

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A NECESSARY CONDITION FOR QUASITRIANGULARITY

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ABSTRACT. In this note we prove that if T is a quasitriangular operator then $\Lambda(T+K)=\Pi(T+K)$ for all compact operators K .

A bounded linear operator T on a Hilbert space $\mathcal{H}$ is called *quasitriangular* if $\liminf_{P \in \mathcal{P}} \|PTP - TP\| = 0$ where $\mathcal{P}$ is the directed set of all finite rank projections in $\mathcal{L}(\mathcal{H})$ under the usual ordering [3]. $\Lambda(T)$, $\Pi_0(T)$, and $\Pi(T)$ will denote the spectrum, point spectrum and approximate point spectrum of T respectively.

THEOREM. *If T is quasitriangular then $\Lambda(T+K)=\Pi(T+K)$ for all compact operators K .*

PROOF. Since T quasitriangular implies $T+K$ is quasitriangular [3], we need only prove that $\Lambda(T)=\Pi(T)$. Suppose that $\Lambda(T) \neq \Pi(T)$ and $\lambda \in \Lambda(T) \setminus \Pi(T)$. Since $\Lambda(T)=\Pi(T) \cup \Pi_0(T^*)^*$ [2, Problem 58], $\lambda^* \in \Pi_0(T^*)$. Hence $T-\lambda$ is bounded below and its adjoint has nontrivial null space. Thus $T-\lambda$ satisfies Lemma 2.1 in [1], and so $T-\lambda$ and hence T is not quasitriangular, proving the Theorem.

REMARKS. 1. It is natural to ask whether the converse to the Theorem is also true, since $\Lambda(T+K_0) \neq \Pi(T+K_0)$ for some compact K_0 is the only known criterion for proving nonquasitriangularity and since $\{T: \Lambda(T+K)=\Pi(T+K) \text{ all compact } K\}$ is also uniformly closed.

2. It is easy to construct for any two compact subsets L and M of the plane satisfying $\partial M \subseteq L \subseteq M$ an (in fact, subnormal) operator S satisfying $\Lambda(S)=M$, and $\Pi(S)=L$. In this way, one can construct nonquasitriangular operators for which one cannot decide about the square.

Received by the editors July 1, 1971.

AMS 1969 subject classifications. Primary 4710, 4745.

Key words and phrases. Quasitriangular operator, compact operator, approximate point spectrum.

¹ This research was partially supported by the National Science Foundation.

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