CONTINUOUS FUNCTIONS FROM A CONNECTED LOCALLY CONNECTED SPACE INTO A CONNECTED SPACE WITH A DISPERSION POINT

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ABSTRACT. For T_2 spaces, it is shown that any continuous function from a connected locally connected space into a connected space with a dispersion point is a constant.

The purpose of this note is to present a result of connected spaces with a dispersion point which seems to have gone unnoticed or is part of the folklore of mathematics, and then we would like to make a remark. There are many examples of connected spaces with a dispersion point, a classic one given by Knaster and Kuratowski [1] and for others, see [2], [3], [4]. For terminology, see [6]. All spaces are T_2 .

THEOREM. *Any continuous function from a connected locally connected space into a connected space with a dispersion point is a constant.*

PROOF. Suppose Y is a connected space with a dispersion point p and f is a continuous function from the locally connected space X into Y .

Assume that f is not constant. This means that $f(X)$ is a nondegenerate connected subset of Y and therefore contains p .

By assumption $f(X) - \{p\} \neq \emptyset$, and therefore the set $G = f^{-1}(Y - \{p\})$ is not empty and different from X . Denote by U any component of G . Since X is locally connected and G open, then U is open. We shall show that U is closed (this will contradict the assumption of connectedness of X).

Since U is connected, so is $f(U)$, and since $f(U) \subset f(G) \subset Y - \{p\}$, we have $p \notin f(U)$. It follows that $f(U)$ is a single point $q \neq p$.

Now to show that U is closed, let $x \in \bar{U}$. Hence $f(x) \in \text{Cl}(f(U))$. But, since $f(U) = \{q\}$, we have $\text{Cl}(f(U)) = \{q\}$ and hence $f(x) = q \neq p$, i.e. $x \in f^{-1}(Y - \{p\}) = G$.

Denote by W the component of G which contains x . Thus $x \in \bar{U} \cap W$ and it follows that $W = U$ since W and U are open sets, either disjoint or identical. Hence $x \in U$. Thus U is closed.

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REMARK. In the original manuscript presented to the editors, we posed the question: Does there exist a nondegenerate connected space Y without a dispersion point such that any continuous function from a connected locally connected space into Y is constant? By methods similar to the above, Mr. J. Krasinkiewicz has shown that a widely connected space has the required property. A connected space Y is said to be widely connected if each nondegenerate connected subset of Y is dense in Y . P. M. Swingle [5] has shown the existence of such a space in the plane.

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