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$\Lambda(p)$ SETS AND SIDON SETS

SAMUEL E. EBENSTEIN

In this note we present a short proof of the following theorem due to Robert E. Edwards, Edwin Hewitt and Kenneth A. Ross [**3**]:

THEOREM. *If Γ is an infinite (discrete) abelian group there exists $E\subset\Gamma$ so that E is $\Lambda(p)$ for all $p<\infty$, but E is not Sidon.*

Our proof is based on a theorem of Aline Bonami [**1**, Theorem 5, p. 359]. We refer to [**4**, §37] for the definition of the terms $\Lambda(p)$ set and Sidon set.

LEMMA 1. *If $\Gamma_0\subset\Gamma$ is a subgroup and $E\subset\Gamma_0$, then E is $\Lambda(p)$ [Sidon] in Γ_0 if and only if E is $\Lambda(p)$ [Sidon] in Γ.*

PROOF. Clear.

LEMMA 2. *If Γ is an infinite abelian group then Γ contains a subgroup Γ_0 isomorphic with one of the following:*

(i) Z;
(ii) $Z(q^\infty)$ *for some prime* q;
(iii) $\sum_{n=1}^{\infty} Z(q_n)$ *where q_n is an increasing sequence of primes*;
(iv) $\sum_{n=1}^{\infty} Z(q)$ *for some prime* q.

PROOF. See [**3**, Theorem 2.3].

The following lemma is a special case of [**2**, Lemma 1, p. 590].

LEMMA 3. *If E is Sidon in Γ and $W\subset\Gamma$ is infinite then $W+W\not\subset E$.*

DEFINITION. If E is a countable subset of Γ let $\gamma_1,\gamma_2,\gamma_3,\cdots$ be an enumeration of the elements of E. For $\gamma\in\Gamma$ and s a positive integer, let $R_s(E,\gamma)$ be the number of representations of γ of the form

$$\gamma=\pm\gamma_{n_1}\pm\gamma_{n_2}\pm\cdots\pm\gamma_{n_s}\qquad(n_1<n_2<\cdots<n_s).$$

The following theorem is a special case of [**1**, Theorem 5, p. 359].

THEOREM 4. *If $E\subset\Gamma$ and $R_s(E,0)\leqq B^s$ for some $B\geqq 0$ and all s, then $E+E$ is $\Lambda(p)$ for all $p<\infty$.*

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PROOF OF THEOREM. By Lemma 3 and Theorem 4 it is sufficient to find an infinite set E so that $R_s(E, 0)=0$, and let $F=E+E$.

If Γ contains an infinite independent set E, then clearly $R_s(E, 0)=0$. If Γ contains no infinite independent set then either $Z\subset\Gamma$, or $Z(q^\infty)\subset\Gamma$ for some prime q. If $Z\subset\Gamma$ let $E=\{3^n\}_{n=1}^\infty$. Then the lacunarity of E implies that $R_s(E, 0)=0$. If $Z(q^\infty)\subset\Gamma$ let $E=\{\exp(2\pi i/q^n)\}_{n=1}^\infty$. (Here we are considering $Z(q^\infty)$ as a subgroup of T.) Then a simple computation shows that $R_s(E, 0)=0$.

BIBLIOGRAPHY

1. Aline Bonami, *Étude des coefficients de Fourier des fonctions de $L^p(G)$*, Ann. Inst. Fourier (Grenoble) **20** (1970), fasc. 2, 335–402.

2. Myriam Déchamps-Gondim, *Compacts associés à un ensemble de Sidon*, C. R. Acad. Sci. Paris Sér. A-B **271** (1970), A590–A592. MR **42** #6526.

3. Robert E. Edwards, Edwin Hewitt and Kenneth A. Ross, *Lacunarity for compact groups*. I, Indiana Univ. Math. J. **21** (1972), 787–806.

4. Edwin Hewitt and Kenneth A. Ross, *Abstract harmonic analysis*. Vol. II: *Structure and analysis for compact groups analysis on locally compact Abelian groups*, Die Grundlehren der math. Wissenschaften, Band 152, Springer-Verlag, Berlin and New York, 1970. MR **41** #7378.

DEPARTMENT OF MATHEMATICS, WAYNE STATE UNIVERSITY, DETROIT, MICHIGAN 48202