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THE NONEXISTENCE OF COMPLEX HAAR SYSTEMS ON NONPLANAR LOCALLY CONNECTED SPACES

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Let X be a compact Hausdorff space and $C(X)$ be the complex linear space of complex valued continuous functions defined on X. An n-dimensional subspace L of $C(X)$, $n \geqq 2$, is called a *complex Haar system* on X if every nonzero member of L has at most $n-1$ zeros. The purpose of this note is to prove

THEOREM 1. *If X is locally connected, then a necessary and sufficient condition for the existence of a complex Haar system of X is that X be imbeddable in the plane.*

This affirms a conjecture of J. Overdeck and generalizes the theorem of Schoenberg and Yang [3] in which X was assumed to be finite polyhedral. To see how their proof extends to the locally connected case, let S^2 be the 2-sphere, K_5 and $K_{3,3}$ be the primitive skew curves (i.e. the complete graph on 5 vertices and the houses and wells configuration), and C_1 and C_2 be the Claytor curves as described on the first page of [1].

THEOREM 2 (CLAYTOR [1]). *If X is a nonplanar Peano continuum, then X contains a subspace homeomorphic to one of S^2, K_5, $K_{3,3}$, C_1, or C_2.*

Since a complex Haar system on X induces one on each of its subspaces, it suffices to show that none of these five spaces admits a complex Haar system. This was done in [3] for S^2, K_5, and $K_{3,3}$. Now observe that C_j contains a nonempty open set U_j such that $C_j - U_j$ is homeomorphic to C_j, $j=1$ or 2. If there were a complex Haar system on C_j, then $C_j - U_j$ would be imbeddable in the plane by Lemma 1 of [2]; but this is impossible since $C_j - U_j$ is homeomorphic to C_j. This proves necessity, and the sufficiency is obvious.

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