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PHYSICAL STATES ON A C^*-ALGEBRA

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ABSTRACT. When a locally compact Hausdorff space X is totally disconnected, any doubly generated subalgebra of (real) $C_0(X)$ is singly generated.

We have noted that the proof of Lemma 9 of [**1**] is incomplete. The gap seems to be quite difficult to fill, so Theorem 1 of [**1**] must be considered as open. We give herein a simple argument which fills that gap for a large class of C^*-algebras.

THEOREM. *Let X be a totally disconnected locally compact Hausdorff space and $C_0(X)$ the algebra of all real-valued continuous functions on X which vanish at infinity. For any $f, g \in C_0(X)$ there exists $h \in C_0(X)$ such that f and g lie in the closed (sup norm) algebra generated by h. (Hence any quasi-linear* [**1**] *functional on $C_0(X)$ is linear.)*

PROOF. Since X is totally disconnected, the Stone-Weierstrass theorem (or a direct construction) shows that the set of all finite linear combinations of idempotents is dense in $C_0(X)$. For each $n=1, 2, \cdots$, there exists a finite family $\{p_i^n\}_{i=1}^{k_n}$ of idempotents and coefficients $\{\alpha_i^n\}$ such that $\| \sum_{i=1}^{k_n} \alpha_i^n p_i^n - f \| < 1/n$.

Consequently $\bigcup_{n=1}^{\infty} \{p_i^n\}_{i=1}^{k_n}$ is a countable family, and f lies in the closed algebra generated by it. Since a similar family exists for g, we may take their union and get a countable family $\{q_n\}_{n=1}^{\infty}$ of idempotents such that both f and g lie in the closed subalgebra generated by it. By [**2**, pp. 293–294] the closed subalgebra generated by $\{q_n\}_{n=1}^{\infty}$ is generated by a single element h, so we are done.

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