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INDEX OF FREDHOLM OPERATORS

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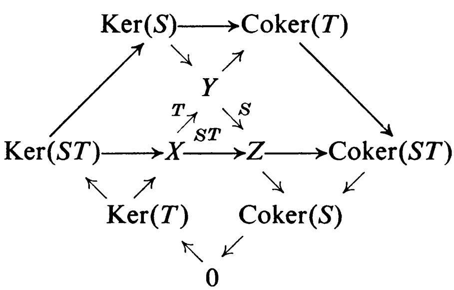
ABSTRACT. Let X, Y, Z be Banach spaces, and let $T:X\to Y$ and $S: Y\to Z$ be Fredholm operators. Let $\operatorname{ind}(T)$ denote the index of T. A short proof is given for the identity $\operatorname{ind}(ST)=\operatorname{ind}(S)+\operatorname{ind}(T)$.

We give a "one-diagram" proof of the following theorem [**3**, Theorem 3, p. 121]. The notation of [**3**] will be used.

THEOREM. *Let X, Y, Z be Banach spaces, and $T:X\to Y$ and $S: Y\to Z$ be Fredholm operators. Then*

$$\operatorname{ind}(ST) = \operatorname{ind}(S) + \operatorname{ind}(T).$$

PROOF. By [**3**, Corollary 3, p. 121], ST is a Fredholm operator. It is easy to verify directly that the outer perimeter sequence in the following commutative diagram of Banach spaces is exact [**1**, p. 25].



Now notice that if $0\to A_0\to A_1\to \cdots \to A_n\to 0$ is an exact sequence of finite dimensional Banach spaces, then

$$\sum_{i=0}^{n}(-1)^i \dim(A_i) = 0.$$

The desired equation follows immediately.

COROLLARY 1. *Let $F:X\to X$ be a Fredholm operator. Then, if for* $n\geqq 0$, $\operatorname{ind}(T^n)=0$, *then* $\operatorname{ind}(T)=0$.

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COROLLARY 2. *Let $K: X \to X$ be a compact operator, and $I: X \to X$ be the identity operator. Then,* $\operatorname{ind}(I+K)=0$.

PROOF. This follows from [2, (11.3.3), p. 321] and Corollary 1.

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