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LOCALIZATION AT INJECTIVES IN COMPLETE CATEGORIES

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ABSTRACT. We consider a complete category $\mathscr{A}$. For each object I of $\mathscr{A}$ we define a functor $Q:\mathscr{A}\to\mathscr{A}$ and obtain a necessary and sufficient condition on I for Q, after restricting its codomain, to become a reflector of $\mathscr{A}$ onto the limit closure of I. In particular, this condition is satisfied if I is injective in $\mathscr{A}$ with regard to equalizers. Among the special cases of such reflectors are: the reflector onto torsion-free divisible objects associated to an injective I in Mod R; the Samuel compactification of a uniform space; the Stone-Čech compactification.

We give a second description of Q in terms of a triple on sets. If I is injective and the functor Q is equivalent to the identity then, under a few extra conditions on $\mathscr{A}$, $\mathscr{A}^{\mathrm{op}}$ is triplable over sets with regard to the functor taking A to $\mathscr{A}(A, I)$.

We recall some notation and definitions. We write $\mathscr{A}(A, B)$ or just (A, B) for the set of all maps from A to B in $\mathscr{A}$. $\mathscr{S}$ denotes the category of sets. The *limit closure* of an object I of $\mathscr{A}$ is the smallest full replete subcategory of $\mathscr{A}$ closed under limits and containing I. I is *injective with regard to* the map $f:A\to B$ if $\mathscr{A}(f, I):\mathscr{A}(B, I)\to\mathscr{A}(A, I)$ is a surjection. We call I *injective* in $\mathscr{A}$ if it is injective with regard to all regular monomorphisms in $\mathscr{A}$. A *regular* monomorphism is a map which happens to be an equalizer.

The object I determines functors

$$\mathscr{A} \xrightarrow{(-,I)} \mathscr{S}^{\mathrm{op}} \xrightarrow{I^{(-)}} \mathscr{A},$$

where $(-, I)$ is a left adjoint of $I^{(-)}$, in view of the natural isomorphism

$$\mathscr{S}^{\mathrm{op}}(\mathscr{A}(A, I), X) = \mathscr{S}(X, \mathscr{A}(A, I)) \cong \mathscr{A}(A, I^X).$$

Thus the composition $S=I^{(-,I)}$ is part of a triple (standard construction) (S, η, μ) on $\mathscr{A}$ (see [9]). For future reference we describe $\eta(A):A\to S(A)$ and also $S(f)$ for any map $f:A\to B$.

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Let u^* denote the map $A \to I^X$ corresponding to $u: X \to (A, I)$, defined by the formula $\forall_{x \in X} \pi_x u^* = u(x)$, where π_x is the canonical projection $I^X \to I$. Then $\eta(A) = 1^*_{(A,I)}$, that is,

$$(*) \qquad \forall_{g \in (A,I)} \pi_g \eta(A) = g.$$

The map $S(f) = I^{(f,I)}: I^{(A,I)} \to I^{(B,I)}$ is given thus

$$(**) \qquad \forall_{f \in (A,B)} \forall_{h \in (B,I)} \pi_h S(f) = \pi_{hf}.$$

Following Fakir [2], we define the functor $Q: \mathscr{A} \to \mathscr{A}$ as the equalizer

$$Q \xrightarrow{\kappa} S \underset{S\eta}{\overset{\eta S}{\rightrightarrows}} S^2.$$

Fakir showed that Q is part of a triple (Q, η_1, μ_1) and that, if $S(\kappa(A))$ is mono for each object A, then Q is idempotent.

Let Fix Q be the full subcategory of $\mathscr{A}$ consisting of all those objects A for which $\eta_1(A): A \to Q(A)$ is an isomorphism. Since $Q(A)$ is only defined up to isomorphism, we can assume that each such $\eta_1(A)$ is the identity map of A. Then Q is idempotent if and only if, by restriction of the codomain to the image, it induces a reflector $\mathscr{A} \to$ Fix Q. The reflection map from A into Fix Q is then $\eta_1(A): A \to Q(A)$, which is defined by the condition $\kappa(A)\eta_1(A) = \eta(A)$.

Before stating our main result, we require two lemmas.

LEMMA 1. *$\kappa(A): Q(A) \to S(A)$ is the joint equalizer of all pairs of maps $S(A) \rightrightarrows I$ which coequalize $\eta(A): A \to S(A)$.*

PROOF. Consider any map $u: S(A) \to I$. Then, by (*), $\pi_u \eta S(A) = u$, and by (**), $\pi_u S\eta(A) = \pi_{u\eta(A)}$. Thus $\kappa(A)$ equalizes all pairs of maps $(u, \pi_{u\eta(A)})$. Now let $v: S(A) \to I$ be such that $u\eta(A) = v\eta(A)$. Then $\kappa(A)$ equalizes (u, v), since

$$u\kappa(A) = \pi_{u\eta(A)}\kappa(A) = \pi_{v\eta(A)}\kappa(A) = v\kappa(A).$$

Conversely, any map which equalizes all (u, v) such that $u\eta(A) = v\eta(A)$ equalizes $(u, \pi_{u\eta(A)})$ in particular, since $\pi_{u\eta(A)}\eta(A) = u\eta(A)$ by (*). Hence it equalizes $\eta S(A)$ and $S\eta(A)$.

LEMMA 2. $I \in$ Fix Q.

PROOF. By (*), $\pi_{1_I}\eta(I) = 1_I$, hence $\eta(I)$ is the equalizer of the pair of maps $(\eta(I)\pi_{1_I}, 1_{S(I)})$. Thus $\eta(I)$ is the joint equalizer of all pairs of maps $I^{(I,I)} \rightrightarrows I^{(I,I)}$ which coequalize $\eta(I)$, and therefore the joint equalizer of all pairs of maps $I^{(I,I)} \rightrightarrows I$ which coequalize $\eta(I)$. In view of Lemma 1,

$\kappa(I)$ is this same equalizer. Hence the unique map $\eta_1(I):I\to Q(I)$ such that $\kappa(I)\eta_1(I)=\eta(I)$ is an isomorphism, and so I is in Fix Q.[1]

THEOREM. *The following statements are equivalent.*

(a) *I is injective with regard to $\kappa(A)$ for each object A in $\mathscr{A}$.*

(b) *Q is idempotent, i.e. becomes a reflector $\mathscr{A}\to\operatorname{Fix} Q$ when its codomain is restricted to its image.*

(c) Fix *Q is the limit closure of I.*

PROOF. (b) and (c) are clearly equivalent, since

$$I\in \operatorname{Fix} Q\subset \text{image of } Q\subset \text{limit closure of } I$$

and a reflective subcategory is limit closed.

(a)$\Rightarrow$(b). We note that if I is injective with regard to a map f then (f, I) is a surjection, that is, a regular mono of $\mathscr{S}^{\mathrm{op}}$. Now $I^{(-)}$, being a right adjoint, preserves limits, so $S(f)=I^{(f,I)}$ is a regular mono of $\mathscr{A}$. Thus it follows from condition (a) that $S(\kappa(A))$ is mono for each A. Fakir's result (Proposition 3 of **[2]**) then shows that Q is idempotent.

(b)$\Rightarrow$(a). We have $A\to_{\eta_1(A)}Q(A)\to_{\kappa(A)}S(A)$ with $\kappa(A)\eta_1(A)=\eta(A)$. Now every map $u:A\to I$ can be extended to $\pi_u:S(A)\to I$, since $\pi_u\eta(A)=u$. Given any map $v:Q(A)\to I$, we let $u=v\eta_1(A)$. Then $\pi_u\kappa(A)\eta_1(A)=u=v\eta_1(A)$. Since $\eta_1(A)$ is a reflection map of A into Fix Q, it follows from Lemma 2 that $\pi_u\kappa(A)=v$. Thus I is injective with regard to $\kappa(A)$.

This completes the proof of the theorem.

DEFINITION. We shall call the object I of A *κ-injective* if it satisfies the equivalent conditions of the theorem. We call Q the *localization* functor determined by I.

COROLLARY. *If I is injective with regard to all equalizers of pairs of maps $I^X\rightrightarrows I^Y$ then Q is (after restricting its codomain) a reflector onto the limit closure of I, which is* Fix Q. *If I is injective in $\mathscr{A}$ then Q takes regular monos of $\mathscr{A}$ to regular monos of* Fix Q.

PROOF. The first statement is obvious. As for the second, let I be injective in $\mathscr{A}$ and $f:A\to B$ a regular mono of $\mathscr{A}$. We already know that $S(f)$ is a regular mono of $\mathscr{A}$ which lies in Fix Q. Since Fix Q is the limit closure of the injective I, Theorem 1 of **[11]** shows that $S(f)$ is a regular mono of Fix Q. Now $\kappa(A):Q(A)\to S(A)$ is also a regular mono of Fix Q and it is easily seen that the composition of two regular monos of Fix Q is a regular mono of Fix Q (since an object A is in Fix Q iff there is an equalizer diagram $A\to I^n\rightrightarrows I^m$). Thus $\kappa(B)Q(f)=S(f)\kappa(A)$ is a regular

[1] The referee has pointed out that Lemma 2 can also be proved directly by observing that π_1 and $S(\pi_1)$ make $I\to S(I)\rightrightarrows S^2(I)$ into a split equalizer diagram.

mono of Fix Q. Since $\kappa(B)$ is mono in Fix Q, $Q(f)$ is also a regular mono of Fix Q.

We remark that if $\mathscr{A}$ is well powered then, by the special adjoint functor theorem, the limit closure of any object I is a reflective subcategory of $\mathscr{A}$.

We shall now consider a number of examples. Further examples in the categories of partially ordered sets, lattices, etc. are being studied by B. Ballinger.

EXAMPLE 1. Take $\mathscr{A}=$ Mod R, where R is an associative ring with unity, and let I be any injective right R-module. We claim that $Q(A)$ is then the usual localization of A, also called the *module of quotients* of A, with respect to the torsion theory determined by I (see [7]).

To prove this, let us provisionally denote the module of quotients by $Q'(A)$. Then $Q'(A)$ is divisible (with respect to I), in the terminology of [7] and may be regarded as a submodule of $S(A)$, since the kernel of $\eta(A)$ is the torsion submodule of A. Now $S(A)$, being a product of copies of I, is torsion-free (with respect to I), hence $S(A)/Q'(A)$ is torsion-free. Since $Q'(A)/\mathrm{Im}\ \eta(A)$ is torsion, it is the torsion submodule of $S(A)/\mathrm{Im}\ \eta(A)$. Thus $s \in Q'(A)$ if and only if

$$\forall_{u:S(A)\to I}(u\eta(A) = 0 \Rightarrow u(s) = 0),$$

that is,

$$\forall_{v,w:S(A)\to I}(v\eta(A) = w\eta(A) \Rightarrow v(s) = w(s)).$$

Therefore $Q'(A)$ is the joint equalizer of all pairs of maps which coequalize $\eta(A)$. Hence, by Lemma 1, $Q'(A)=Q(A)$, as was to be proved.

EXAMPLE 2. Take $\mathscr{A}$ to be the category of topological spaces and let I be the unit interval $[0, 1]$. I is not injective, but it is κ-injective. It is easily seen that $Q(A)$ is the closure in $S(A)$ of the image of $\eta(A)$. Thus the construction of $Q(A)$ is the familiar construction of the Stone-Čech compactification $\beta(A)$, as described by Čech [**1**]. Condition (a) in this example is simply a special case of Tietze's theorem, since $S(A)=I^{(A,I)}$ is normal and $Q(A)$ is a closed subspace.

Of course, there is no reason for β to preserve regular monos, as I is not injective (with regard to all regular monos) in $\mathscr{A}$.[2]

EXAMPLE 3. Take $\mathscr{A}$ to be the category of uniform spaces (not necessarily Hausdorff) and let $I=[0, 1]$. Regular monomorphisms in $\mathscr{A}$ are easily seen to be the same as subspace inclusions. It is known that I is injective with regard to subspace inclusions (see [**6**]).

The reflector Q is the Samuel compactification (see [**12**] or [**5**]), that is, Fix Q consists of all compact Hausdorff uniform spaces.

[2] George Reynolds has observed that one can similarly obtain the real compactification of A by taking I to be the real line.

To see this, we note that $Q(A)$ is easily shown to be the closure in $S(A)$ of the image of $\eta(A)$. Now $S(A)=I^{(A,I)}$ is compact, and therefore so is the closed subspace $Q(A)$. (We observe that the forgetful functor from uniform to topological spaces preserves products.) Conversely, assume that A is compact Hausdorff, and recall that all continuous mappings from a compact Hausdorff uniform space are uniformly continuous. Then $Q(A)$ is the Stone-Čech compactification of A, and hence $\eta_1(A)$ is an isomorphism.

There are many other injectives in $\mathscr{A}$. For example, if M is any metrizable uniform space then (M, I) with the obvious metric is injective [5]. It is not clear what the associated category Fix Q is. It certainly contains only complete spaces. It seems unlikely that there is any M for which Fix Q contains all complete spaces, that is, such that $Q(A)$ is the Hausdorff completion of A.[3]

EXAMPLE 4. Let $\mathscr{A}$ be the category of presheaves on a small category $\mathscr{X}$, that is, the category of all functors $\mathscr{X}^{\mathrm{op}}\rightarrow\mathscr{S}$. Suppose that $\mathscr{X}$ is equipped with a Grothendieck topology, then one can construct a huge injective I whose limit closure is the category of sheaves for the given topology. For any presheaf A, $Q(A)$ is then the associated sheaf.

The story is somewhat different when $\mathscr{A}$ is an elementary topos in the sense of Lawvere and Tierney. Since $\mathscr{A}$ is not necessarily complete, our construction of the triple S does not work. However, an analogous construction does work, and we shall sketch it briefly.

Let I be any injective in $\mathscr{A}$. Since $\mathscr{A}$ is a cartesian closed category, we can obtain a triple S on $\mathscr{A}$ from the selfadjoint functor $\mathscr{A}\rightarrow\mathscr{A}^{\mathrm{op}}$ taking A to I^A, with $S(A)=I^{I^A}$. Fakir's construction applied to S then gives an idempotent triple Q. However, Fix Q is not the limit closure of I; it must also be closed under internal powers. It turns out that Q preserves all finite limits.

The work of Lawvere and Tierney suggests which injectives I one should single out for consideration: Let j be any Heyting endomorphism of the subobject classifier Ω, then take $I=\Omega_j$, the retract of Ω determined by j. Fix Q will then be the category of j-sheaves. (For definitions see the discussion of the work of Lawvere and Tierney in [3].)

We plan to elaborate the details of this example in a sequel to the present paper.

We shall give another interpretation of the localization functor Q

[3] Indeed, George Reynolds has observed that the completion functor in uniform spaces cannot be obtained by our method from a single uniform space I, because any such space has a cardinal number associated with it which is preserved by products and subspace formation, namely the smallest infinite cardinal such that every uniform cover has a refinement less than it.

with the help of the Eilenberg-Moore category of a triple. It will be convenient here to replace the complete category $\mathscr{A}$ by its opposite, the cocomplete category $\mathscr{B}=\mathscr{A}^{\mathrm{op}}$. We consider an object P of $\mathscr{B}$ which is κ-projective, that is, κ-injective as an object of $\mathscr{A}$.

The functor $U=(P, -):\mathscr{B}\to\mathscr{S}$ has a left adjoint F, where $F(X)=\sum_{x\in X}P$, let us say with adjunction $\eta:\mathrm{id}\to UF$ and coadjunction $\varepsilon:FU\to\mathrm{id}$. The triple $(UF, \eta, U\varepsilon F)$ on $\mathscr{S}$ gives rise to the Eilenberg-Moore category $\mathscr{S}^{UF}$, whose objects are certain pairs (X, ξ), where $\xi:UF(X)\to X$ in $\mathscr{S}$. One studies the so-called *comparison* functor $K:\mathscr{B}\to\mathscr{S}^{UF}$ given by

$$K(B) = (U(B), U\varepsilon(B)), \qquad K(b) = U(b).$$

This has a left adjoint M (see [**9**, p. 151, Exercise 5]), where $M(X, \xi)$ is the coequalizing object of the pair

$$FUF(X) \underset{\varepsilon F(X)}{\overset{F(\xi)}{\rightrightarrows}} F(X).$$

A simple calculation shows the following:

PROPOSITION 1. *If K is the comparison functor of $(\mathscr{B}, U)$, M its left adjoint, and Q the localization functor on $\mathscr{B}^{\mathrm{op}}$, then $MK=Q$.*

We may also call Q the *colocalization* functor on $\mathscr{B}$.

We consider an interesting special case.

EXAMPLE 5. Take $\mathscr{B}=\mathrm{Mod}\,R$ and let P be any finitely generated κ-projective right R-module. Let E be the ring (P, P), then $(P, -)$ may be considered as a functor $\mathscr{B}\to\mathrm{Mod}\,E$ with left adjoint $(-)\otimes_E P$. Since P is finitely generated, $(P, -)$ takes sums in $\mathscr{B}$ to sums in $\mathrm{Mod}\,E$. Let U_E denote the forgetful functor $\mathrm{Mod}\,E\to\mathscr{S}$. Then the triple UF is given by

$$UF(X) = U_E\left(P, \sum_X P\right) \cong U_E\left(\sum_X E\right).$$

Now this is the triple associated to $\mathrm{Mod}\,E$, so $\mathscr{S}^{UF}=\mathrm{Mod}\,E$. The functor K is clearly $(P, -)$, hence $M=(-)\otimes_E P$, and we have

$$Q(B) = (P, B)\otimes_E P.$$

This formula actually holds in a more general situation. Let $\mathscr{B}$ be any cocomplete abelian category and P any κ-projective object which is *small* in the sense that $(P, -):\mathscr{B}\to\mathrm{Mod}\,E$ preserves sums.

The smallness of P will follow easily if it is assumed to be *finitely generated* in the sense of [**4**], that is, that $(P, -)$ preserves directed colimits of monomorphisms. In this definition it makes no difference whether $(P, -)$ is considered as a functor into $\mathrm{Mod}\,E$ or into $\mathscr{S}$.[4]

[4] We are indebted to the referee for criticizing the original discussion of this example, in which the condition that P be finitely generated had been overlooked.

PROPOSITION 2. *Let Q be the colocalization functor determined by the κ-projective object P of the cocomplete category $\mathscr{B}$. Then the following conditions are equivalent.*

(1) *Q is canonically isomorphic to the identity, that is, η_1 is a natural isomorphism.*

(2) *P is a regular generator in the sense that for each object B of $\mathscr{B}$ there is a regular epi from some multiple of P to B.*

(3) *For each object B of $\mathscr{B}$ there is a coequalizer diagram $mP\rightrightarrows nP\to B$, for some cardinal numbers m and n.*

(4) *$\mathscr{B}$ is the colimit closure of P.*

PROOF. The implication (4)$\Rightarrow$(1) is an immediate consequence of our theorem. For, since P is κ-projective, the theorem tells us that the colimit of P is Fix Q, and by (4) this is $\mathscr{B}$, so that (1) holds.

The implications (1)$\Rightarrow$(2)$\Rightarrow$(3)$\Rightarrow$(4) hold even without the assumption that P is κ-projective. The first and last implications are clear; we shall prove that (2)$\Rightarrow$(3). Assume (2) and let B be any object of $\mathscr{B}$. Then there is a coequalizer diagram $B'\rightrightarrows nP\to B$ and a regular epi $mP\to B'$, hence a coequalizer diagram $mP\rightrightarrows nP\to B$.

Clearly (1) implies that P is κ-projective. Hence (1) asserts that P is a *κ-projective regular generator.*

We present the following variant of Linton's theorem (see [**8**, p. 88]).

PROPOSITION 3. *Let P be an object of the category $\mathscr{B}$ and $U=\mathscr{B}(P, -)$, then $(\mathscr{B}, U)$ is varietal (triplable) if and only if*

(1) *$\mathscr{B}$ is cocomplete and has kernel pairs,*

(2) *P is a projective regular generator,*

(3) *every equivalence relation in $\mathscr{B}$ is a kernel pair.*

A pair of maps $R\rightrightarrows A$ in $\mathscr{B}$ is here called an *equivalence relation* if $\mathscr{B}(B, R)\rightrightarrows\mathscr{B}(B, A)$ is an equivalence relation in $\mathscr{S}$ for every object B of $\mathscr{B}$.[5]

PROOF. *Necessity of conditions.* (1) and (3) are well-known properties of varietal categories, and (2) follows from Proposition 1.

Sufficiency of conditions. We verify Linton's conditions *FIT* ([**8**, p. 88]). We know that U has a left adjoint and that $(FIT)_0$ holds, that is, $\mathscr{B}$ has kernel pairs and coequalizers. $(FIT)_1$ says that $\pi: A\to B$ in $\mathscr{B}$ is a regular epi if and only if $U(\pi)$ is a surjection.

Since P is projective, U takes regular epis to surjections. Conversely, if $U(\pi)$ is a surjection, then $FU(\pi)$ is a regular epi, since F preserves

[5] Both M. Barr and C. Mulvey have informed us that they have obtained essentially the same version of Linton's theorem.

limits. Now consider the following commutative diagram:

$$\begin{tikzcd}
FU(A) \arrow[r, "FU(\pi)"] \arrow[d, "\varepsilon(A)"'] & FU(B) \arrow[d, "\varepsilon(B)"] \\
A \arrow[r, "\pi"'] & B
\end{tikzcd}$$

Since $Q \simeq \mathrm{id}$ canonically, $\varepsilon(A)$ and $\varepsilon(B)$ are regular epis. Now the composition $\varepsilon(B)FU(\pi)$ of two regular epis is a regular epi, since $\mathscr{B}$ is the limit closure of the projective P (see **[11**, Lemma 2.2]). Since $\pi\varepsilon(A)$ is a regular epi and $\varepsilon(A)$ is epi, π is a regular epi.

$(FIT)_2$ says that $p_1, p_2 : R \rightrightarrows A$ is a kernel pair in $\mathscr{B}$ if and only if $U(p_1)$, $U(p_2): U(R) \rightrightarrows U(A)$ is a kernel pair in $\mathscr{S}$. The "only if" part is obvious, since U is representable. Consider the full subcategory $\mathscr{C}$ of $\mathscr{B}$ consisting of those objects B for which the representable functor $(B, -)$ takes (p_1, p_2) into a kernel pair. $\mathscr{C}$ contains P and is obviously replete. It is easily seen to be closed under colimits, and since $\mathscr{B}$ is the colimit closure of P, $\mathscr{C} = \mathscr{B}$.

COROLLARY 1. *Let $\mathscr{A}$ be a complete Abelian category with an injective cogenerator I. Then $\mathscr{A}^{\mathrm{op}}$ is varietal with respect to the functor $\mathscr{A}(-, I)$.*

PROOF. $\mathscr{B} = \mathscr{A}^{\mathrm{op}}$ satisfies the conditions of Proposition 3 with $P = I$. Indeed, conditions (1) and (2) are obvious. To prove (3) we observe that in Mod R every equivalence relation is a kernel pair. By Mitchell's embedding theorem, the same is true in any Abelian category, hence in the opposite of an Abelian category.

EXAMPLE 6. The opposite of any Grothendieck category $\mathscr{A}$ is varietal, as $\mathscr{A}$ contains an injective cogenerator. Oberst **[10]** has also described $\mathscr{A}^{\mathrm{op}}$ as a concrete category with the forgetful functor $\mathscr{A}(-, I)$. However, the structure he defines involves topology and is not obviously varietal.

COROLLARY 2. *Let Q be the colocalization functor associated with the projective object P of the cocomplete category $\mathscr{B}$. Assume $\mathscr{B}$ has kernel pairs and all equivalence relations in the category* Fix Q *are kernel pairs. Then* Fix Q *is varietal with respect to the functor* $(P, -): \mathrm{Fix}\ Q \to \mathscr{S}$.

PROOF. Fix Q satisfies conditions (1) to (3) of Proposition 3.

Example 6 could also have been deduced from Corollary 2 by means of the Gabriel-Popescu theorem.

EXAMPLE 7. Let $\mathscr{A}$ be the category of all set-valued sheaves with respect to a Grothendieck topology. Then $\mathscr{A}^{\mathrm{op}}$ is varietal.

A proof will be given in a sequel to this paper.

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