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Ming-Jung Lee, *Homotopy for functors*, Proc. Amer. Math. Soc. **36** (1972), 571–577.

Proposition 4 in [**3**] is incorrect. Here we state and prove a new version of this proposition. We also modify the proof of Proposition 5 in [**3**] so as to rely on the new Proposition 4 instead of the old one. The author is grateful to Ellis Cooper and Rudolf Fritsch for pointing out the mistake, especially Rudolf Fritsch for suggesting the correction and further so carefully listing all the misprints.

The mistake was caused by overlooking the possibility of degenerate α's in the equivalence relation in the definition of *Sd* [**2**]. It is necessary to create more degeneracies for the equivalence relation to kill. We need the functor P of [**1**], or equivalently the functor $Q \circ F$ of [**4**], which is a functor from the category of simplicial sets to itself and defined as follows. (For convenience, we follow the definition as in [**4**].)

For any simplicial set K, PK is the simplicial set with $PK_n = \{(\sigma, \beta) | \beta\colon [n] \to [\dim \sigma]$ is in Δ and surjective$\}$ and for any α in Δ with range $[n]$, $(\sigma, \beta)\alpha = (\sigma\varphi_1, \varphi_2)$ where $\varphi_1 \circ \varphi_2 = \beta \circ \alpha$ is the unique decomposition of $\beta \circ \alpha$ with φ_1 injective and φ_2 surjective. The correct version of Proposition 4 is as follows.

Proposition 4. *For any simplicial set K, $M\Lambda K$ is naturally isomorphic to $Sd \circ PK$ where Sd is the subdivision functor of* [**2**].

Proof. In order to relate the notations, we recall from [**2**] first that $(Sd \circ PK)_p = \{((\sigma, \beta), \xi) | (\sigma, \beta) \in PK,\ \ \xi = [x_p, x_{p-1}, \cdots, x_0]$ with range $x_p = (\sigma, \beta)$ and $x_p \circ \cdots \circ x_0$ is composable$\}/\sim$, where "$\sim$" is the equivalence relation generated by letting $((\sigma, \beta), \xi) \sim ((\tau, \gamma), \rho)$ if there is an α in Δ with $(\tau, \gamma)\alpha = (\sigma, \beta)$ and $\Delta'(\alpha)(\xi) = \rho$. (Note that x_i's are morphisms in ΛPK and they can be also considered as face maps in Δ.) Recall also the map $\Delta'(\alpha)$ carries $\xi = [x_p, \cdots, x_0]$ to $[y_p, y_{p-1}, \cdots, y_0]$ where y_i is defined inductively to be the unique face map in Δ for which there exists an epimorphism

$$\gamma_i\colon [\dim \text{ domain } x_i] \to [\dim \text{ domain } y_i]$$

such that the following diagram commutes.

$$\begin{array}{ccc} [\dim \text{domain } x_i] & \xrightarrow{x_p \circ \cdots \circ x_i} & [n] = \text{domain } \alpha \\ \downarrow{\scriptstyle y_i} & & \downarrow{\scriptstyle \alpha} \\ [\dim \text{domain } y_i] & \xrightarrow{y_p \circ \cdots \circ y_i} & [m] = \text{range } \alpha \end{array}$$

(In ΛPK, Range $y_i=(\tau, \gamma)y_p \cdots y_{i+1}$ and domain $y_i=(\tau, \gamma)y_p \cdots y_i$.)

Define a map $\Phi: Sd \circ PK \to M\Lambda K$ by

$$\Phi_p((\sigma, \beta), \xi) = [\bar{x}_{p-1}, \cdots, \bar{x}_0]$$

where $[\bar{x}_p, \cdots, \bar{x}_0]=\Delta'(\beta)(\xi)$ and $\bar{x}_i$ is now regarded as a morphism in ΛK with range $\bar{x}_i=\sigma\bar{x}_p\bar{x}_{p-1} \cdots \bar{x}_{i+1}$ and domain $x_i=\sigma\bar{x}_p \cdots \bar{x}_i$. Note that $\bar{x}_p$ is not present in $\Phi_p((\sigma, \beta), \xi)$. To show Φ is well defined, we suppose $((\sigma, \beta), \xi)\sim((\tau, \gamma), \rho)$, or for simplicity we may assume there is a morphism α in Δ with $(\tau, \gamma)\alpha=(\sigma, \beta)$ and $\Delta'(\alpha)(\xi)=\rho$. Decompose $\gamma\alpha$ into $\varphi_1 \cdot \varphi_2$ with φ_1 injective and φ_2 surjective; then we have $\tau\varphi_1=\sigma$, $\varphi_2=\beta$ and further

$$\begin{aligned} \Delta'(\varphi_1)\Delta'(\beta)(\xi) &= \Delta'(\varphi_1\varphi_2)(\xi) = \Delta'(\gamma\alpha)(\xi) \\ &= \Delta'(\gamma)\Delta'(\alpha)(\xi) = \Delta'(\gamma)(\rho), \end{aligned}$$

i.e. $\Delta'(\varphi_1)[\bar{x}_p, \cdots, \bar{x}_0]=[\bar{y}_p, \cdots, \bar{y}_0]$ where $\Delta'(\beta)(\xi)=[\bar{x}_p, \cdots, \bar{x}_0]$ and $\Delta'(\gamma)(\rho)=[\bar{y}_p, \cdots, \bar{y}_0]$. It follows from the injectivity of φ_1 and a simple argument of maps that $\varphi_1\bar{x}_x=\bar{y}_p$ and $[\bar{x}_{p-1}, \cdots, \bar{x}_0]=[\bar{y}_{p-1}, \cdots, \bar{y}_0]$ in Δ. Moreover, since $\tau\bar{y}_p=\tau\varphi_1\bar{x}_p=\sigma\bar{x}_p$, we conclude $\Phi((\sigma, \beta), \xi)= [\bar{x}_{p-1}, \cdots, \bar{x}_0]=[\bar{y}_{p-1}, \cdots, \bar{y}_0]=\Phi((\tau, \gamma), \rho)$ in $M\Lambda K_p$. Conversely, to show Φ is one-to-one, we assume $\Phi((\sigma, \beta), \xi)=\Phi((\tau, \gamma), \rho)$ in $M\Lambda K_p$. Then we have $\sigma\bar{x}_p=\tau\bar{y}_p$ and $[\bar{x}_{p-1}, \cdots, \bar{x}_0]=[\bar{y}_{p-1}, \cdots, \bar{y}_0]$ in Δ. Finally,

$$\begin{aligned} ((\tau, \gamma), \rho) &\sim ((\tau, \text{id}), \Delta'(\gamma)\rho) \sim ((\tau\bar{y}_p, \text{id}), \tilde{\rho}) \\ &= ((\sigma\bar{x}_p, \text{id}), \tilde{\xi}) \sim ((\sigma, \text{id}), \Delta'(\beta)\xi) \sim ((\sigma, \beta), \xi) \end{aligned}$$

where $\tilde{\rho}=[\text{id}, \bar{y}_{p-1}, \cdots, \bar{y}_0]$ and $\tilde{\xi}=(\text{id}, \bar{x}_{p-1}, \cdots, \bar{x}_0)$. Q.E.D.

The proof of Proposition 5 is almost unchanged, but one has to add that there is a natural transformation $r: P \to 1$ which also becomes a weak homotopy equivalence after geometric realization (see **[1]** or **[4]**). The Remark on p. 575 of **[3]** should be withdrawn because of lack of proof. The answer to Question 2 in **[3]** is positive if the range of the considered simplicial maps is a Kan set (proof due to R. Fritsch) because a Kan set is a retract of the singular set of its geometric realization (see K. Lamotke **[5]**). Finally we wish to remark that the converse of Proposition 7 in **[3]** is also true (see D. Anderson **[6]**).

List of misprints:

p. 572 6th line from the bottom: "$\partial_0[x_0]=(u_1)$" not "$\partial_0[x_0]=(u_0)$".
5th line from the bottom: "$\partial_1[x_0]=(u_0)$" not "$\partial_1[x_0]=(u_1)$".

p. 573 7th line in section (b): "$x^0_{j-1}\circ x^0_{j-2}$" not "x^0_{j-1}, x^0_{j-2}".

p. 574 9th, 13th, 15th line: "$M\Lambda(K)$" not "$S\Lambda(K)$".

p. 575 1st to 2nd line: "$y_i\varphi_i(x)=\varphi_{i+1}(x)y_i$ or $y_i\varphi_{i+1}(x)=\varphi_i(x)y_i$" not only "$y_i\circ\varphi_i(x)=\varphi_{i+1}\circ y_i$".
3rd line: "$y^{-1}\varphi'(x)y$" not "$y^{-1}\varphi(x)y$".

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