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ADDENDUM TO "THE SCHUR MULTIPLICATOR OF METACYCLIC GROUPS"

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ABSTRACT. Every metacyclic group has a metacyclic group with trivial multiplicator among its representing groups.

In [1] one of us computed the Schur multiplicator of the typical (finite) metacyclic group

$$(1) \quad G(M, N, r, \lambda) = (a, b : a^M = 1, b \cdot a \cdot b^{-1} = a^r, b^N = a^{M\lambda/(M,r-1)})$$

where $r^N \equiv 1 \bmod M$ and λ divides

$$(2) \qquad h(M, N, r) = M^{-1} \cdot (M, r-1) \cdot (M, 1 + r + \cdots + r^{N-1}).$$

A Schur group is a group with trivial Schur multiplicator. The following argument uses Schur's theory [2] of representing groups (=Darstellungsgruppen) for an alternate computation of the Schur multiplicator $H_2G(M, N, r, \lambda)$.

THEOREM. *Every metacyclic group G has a metacyclic Schur group $\tilde{G}$ among its representing groups. If $G = G(M, N, r, \lambda)$ with $r^N \equiv 1 \bmod M$ and $\lambda | h(M, N, r)$, then $s \equiv r \bmod M$ can be found such that*

(i) *$s^N \equiv 1 \bmod M \cdot \lambda$ and thus $\tilde{G} = G(M \cdot \lambda, N, s, 1)$ is defined and*

(ii) *there exists a central stem extension*

$$Z_\lambda \rightarrowtail G(M \cdot \lambda, N, s, 1) \twoheadrightarrow G(M, N, r, \lambda)$$

which exhibits $\tilde{G}$ as a representing group of G.

Hence $H_2G(M, N, r, \lambda) \approx Z_\lambda$.

PROOF. We first find an integer $s \equiv r \bmod M$ such that $(s-1)/(M, s-1)$ is prime to M; any such s can be used in the sequel. Let d be the largest factor of M prime to $(r-1)/(M, r-1)$; set $s = r + d \cdot M$. Clearly

$$h(M, N, s) = h(M, N, r) \quad \text{and} \quad G(M, N, s, \lambda) \approx G(M, N, r, \lambda).$$

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From

$$(M \cdot \lambda, s-1) \mid (M \cdot (M, s-1), s-1)$$
$$= (M, s-1) \cdot (M, (s-1)/(M, s-1)) = (M, s-1)$$

we deduce $(M \cdot \lambda, s-1)=(M, s-1)$.

We conclude from (2) that $M \cdot \lambda | M \cdot h | s^N-1$, hence (i). Let $\tilde{G}=G(M \cdot \lambda, N, s, 1)$ be presented as in (1). The element $c=a^M$ is central in $\tilde{G}$ because $[b, c]=bcb^{-1}c^{-1}=a^{M\cdot(s-1)}=1$. Since $[\tilde{G}, \tilde{G}]$ is generated by $a^{(M\cdot\lambda, s-1)}=a^{(M, s-1)}$, it contains c. Thus the cyclic subgroup $\boldsymbol{Z}_\lambda$ generated by c lies in $[\tilde{G}, \tilde{G}] \cap \operatorname{center}(\tilde{G})$. The factor group $\tilde{G}/\boldsymbol{Z}_\lambda$ is $G(M, N, s, \lambda)\approx G$, which is immediate from the relations.

Recall from [**1**, proof of Theorem 4] that $\tilde{G}$ can be presented with two generators only; hence it is a Schur group. Hence, by Schur [**2**, Satz IV, p. 100], the extension (ii) exhibits $\tilde{G}$ as a representing group of G and the kernel $\boldsymbol{Z}_\lambda$ is isomorphic to the multiplicator of G.

References

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