PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 43, Number 2, April 1974

A RELATION IN $H^*(MO\langle 8\rangle, Z_2)$

V. GIAMBALVO[1]

ABSTRACT. It is shown that $H^*(MO\langle 8\rangle, Z_2)$ does not split as a module over the Steenrod algebra into a direct sum of modules, each having a single generator.

The standard method of computing cobordism groups is to compute the cohomology of the associated Thom spectrum as a module over the Steenrod algebra $\mathscr{A}$, and then apply the Adams spectral sequence. In most cases presently known, this cohomology splits over $\mathscr{A}$ into the direct sum of modules on one generator, and these are fairly accessible to the Adams spectral sequence. This is the case for unoriented, oriented, unitary, SU, and Spin cobordism. In this note the cobordism group associated to the 7-connected covering of BO is discussed. Partial results were obtained in [**2**]. For details of the other cobordism groups see [**1**], [**3**], [**4**].

Let BO be the classifying space for stable vector bundles. For the $(n-1)$-connected covering $BO\langle n\rangle$ of BO, there is a cobordism group $\Omega^{\langle n\rangle}$ whose associated Thom spectrum is the Thom space of the pullback of the canonical bundle over BO. (Actually it is the limit over finite stages.) Since $BO\langle 1\rangle = BO$, $BO\langle 2\rangle = BSO$, and $BO\langle 4\rangle = B\,\mathrm{Spin}$, these give the usual known cobordism groups. For $n>8$, the exotic classes in $H^*(BO\langle n\rangle, Z_2)$ prevent a splitting over the mod 2 Steenrod algebra into a direct sum of modules on one generator. But since $H^*(BO\langle 8\rangle, Z_2)$ is a quotient of $H^*(BO)$, the splitting of $H^*(BO\langle 8\rangle, Z_2)$ remained open. In [**2**] it was shown that a splitting does exist up to dimension 50. The following result shows that this does not extend.

THEOREM. *The submodule $\mathscr{A}U \subset H^*(MO\langle 8\rangle, Z_2)$ generated over the Steenrod algebra by the Thom class U is not a direct summand.*

Received by the editors July 3, 1973.

AMS (MOS) subject classifications (1970). Primary 57D90; Secondary 55G10.

Key words and phrases. $\langle 8\rangle$-cobordism, Steenrod algebra, splitting over Steenrod algebra.

[1] This research was supported in part by the University of Connecticut Computer Center.

PROOF.

$$(Sq^{41}Sq^{14} + Sq^{43}Sq^{12})U + Sq^{29}Sq^{4}Sq^{2}((w_8w_{12} + w_{20})U)$$
$$+ Sq^{13}Sq^{4}Sq^{2}((w_{36} + w_{28}w_8 + w_{22}w_{14} + w_{24}w_{12} + w_{20}w_{10} + w_{12}w_8^3)U) = 0.$$

This relation was obtained by attempting to compute the $\mathscr{A}$ module structure of $H^*(MO\langle 8\rangle, Z_2)$ on the IBM 360 at the University of Connecticut Computer Center.

REFERENCES

1. D. W. Anderson, E. H. Brown, Jr. and F. P. Peterson, *The structure of the Spin cobordism ring*, Ann. of Math. **90** (1969), 157–186.

2. V. Giambalvo, *On* $\langle 8\rangle$ *cobordism*, Illinois J. Math. **15** (1971), 533–541. MR **44** #4757.

3. J. Milnor, *On the cobordism ring* Ω^* *and a complex analogue*. I, Amer. J. Math. **82** (1960), 505–521. MR **22** #9975.

4. R. Thom, *Quelques propriétés globales des variétés differentiables*, Comment. Math. Helv. **28** (1954), 17–86. MR **15**, 890.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CONNECTICUT, STORRS, CONNECTICUT 06268