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A SHORT PROOF OF THE CLASSICAL EDGE OF THE WEDGE THEOREM

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ABSTRACT. By solving the $\bar{\partial}$-equation and using Bochner's theorem on tube domains, one may derive an easy proof of the edge of the wedge theorem in 2 variables.

Let $\boldsymbol{R}^n_+$ and $\boldsymbol{R}^n_-$ denote the (open) positive and negative cones in $\boldsymbol{R}^n$. The classical edge of the wedge theorem states that a function which is analytic on the set $\{z \in \boldsymbol{C}^n : (\operatorname{Re} z_1, \cdots, \operatorname{Re} z_n) \in \boldsymbol{R}^n_+ \cup \boldsymbol{R}^n_-\}$ and continuous where $(\operatorname{Re} z_1, \cdots, \operatorname{Re} z_n) = (0, \cdots, 0)$ can be continued holomorphically to the whole space $\boldsymbol{C}^n$. We prove a slightly weaker version of this.

THEOREM. *Let* $W = \{z \in \boldsymbol{C}^2 : \operatorname{Re} z \in \boldsymbol{R}^2_+ \cup \boldsymbol{R}^2_-\}$. *If* $f \in C^2(\boldsymbol{C}^2)$ *and if* f *is analytic on* W, *then* f *can be continued analytically to* $\boldsymbol{C}^2$.

PROOF. Define two (0, 1)-forms:

$$\begin{aligned} h_1(z) &= \bar{\partial} f(z), \quad && \operatorname{Re} z_1 > 0, \\ &= 0, && \text{otherwise}; \\ h_2(z) &= \bar{\partial} f(z) - h_1(z). \end{aligned}$$

Observe that $\partial f / \partial \bar{z}_j$ and $\operatorname{grad}(\partial f / \partial \bar{z}_j)$ vanish on W for $j = 1, 2$ and therefore on the set $\{z : \operatorname{Re} z_1 = 0\}$. Thus, the forms h_j are continuously differentiable, and $\bar{\partial} h_j = 0$ for $j = 1, 2$.

Let g_j be functions such that $\bar{\partial} g_j = h_j$ for $j = 1, 2$. Then g_1 is analytic on the set $W_1 = \{(z_1, z_2) : \operatorname{Re} z_1 < 0 \text{ or } \operatorname{Re} z_2 > 0\}$, and it follows from Bochner's theorem on tube domains that there is an entire analytic continuation G_1 of g_1. Similarly there is an entire function G_2 continuing the function g_2 from the set $\{z : \operatorname{Re} z_1 > 0 \text{ or } \operatorname{Re} z_2 < 0\}$. Thus $F = f + (G_1 - g_1) + (G_2 - g_2)$ is the desired analytic continuation of f.

REMARK. The edge of the wedge theorem in $\boldsymbol{C}^n$ $(n > 2)$ may be deduced from the two-dimensional result along the following lines. If a function satisfies the hypotheses of the above Theorem in $\boldsymbol{C}^n$, then it also satisfies them on each complex 2-plane through $(1, 1, \cdots, 1)$ and the origin.

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By the Theorem for C^2, the function can be continued analytically to each of these complex 2-planes. The extension defined in this way can be shown to be analytic in C^n.

References

1. L. Hörmander, *An introduction to complex analysis in several variables*, Van Nostrand, Princeton, N.J., 1966. MR **34** #2933.

2. W. Rudin, *Lectures on the edge-of-the-wedge theorem*, CBMS Regional Conference Series in Math., no. 6, Amer. Math. Soc., Providence, R.I., 1971.

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