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THE BRAUER GROUP OF AN AMITSUR FIELD. II

CHAN-NAN CHANG

ABSTRACT. The structure of the Brauer group over an Amitsur field with characteristic 0 has been determined in [5]. In this paper we extend these results to the Amitsur field with characteristic $p > 0$.

1. **Amitsur fields.** Let K be a field and $K\{t\}$ be the field of all formal power series $f\{t\} = \Sigma_{v \geq r} a_v t^v$, $a_v \in K$, $r > -\infty$. Let $t_1, \cdots, t_m$ be a set of indeterminates over K. Define successively, $K_0 = K$, $K_1 = K_0\{t_1\}, \cdots, K_m = K_{m-1}\{t_m\}$. K_m is called an Amitsur field of m-indeterminates (or an Amitsur field) if K is algebraically closed. Throughout this paper we assume K'_m is an Amitsur field with characteristic $p > 0$.

The order function o_m and the valuation $|\ |_m$ on K_m are defined as in [5]. Similarly denote the ring of integers by $\mathfrak{O}_m$, its unique maximal ideal by $\mathfrak{P}_m$ and the residue class field $\bar{K}_m = \mathfrak{O}_m/\mathfrak{P}_m \cong K_{m-1}$.

2. **The character group of** K_m. Let A be an abelian torsion group and let q be a prime. Let $A(q)$ denote the q-primary component of A, and A_q the subgroup of the elements in A which have orders relatively prime to q. Let g: $A \to B$ be a homomorphism of abelian torsion groups. g is said to be *q-injective, q-surjective, q-isomorphism* if g is injective, surjective, bijective, respectively, modulo q-primary components. Denote a q-isomorphism by $\cong_q$.

Let F be a field. Denote the algebraic closure of F by F^c and the Galois group of F^c over F by $G(F)$. Let L be an algebraic extension of F. We use L_q to denote the field of all elements $x \in L$ with $[F(x)\colon F]$ being relatively prime to q.

A continuous homomorphism (Krull topology) of $G(F)$ into Q/Z (the rationals mod 1) is called a character over F. We use $\chi(F)$ to denote the group of all characters over F. In this section we determine $(\chi(K_m))_p$ by using induction on m.

Let $\hat{Z}$ be the Artin group which is the completion of Z with the topology of subgroups of finite index.

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2.1 **Lemma.** *Let* $f\{t\} = 1 + \Sigma_{j \geq 1} \alpha_j t_m^j \in K_{m-1}[[t_m]]$. *Then* $f\{t\} \in K_m^n = \{a^n \mid a \in K_m\}$ *for every positive integer* n *relatively prime to* p.

Proof. By direct computation. Q.E.D.

2.2 **Proposition.** *Let* L *be a finite extension of* K_m *with* $[L: K_m]$ *being prime to* p. *Then there exist* $T \in L$ *with* $T^r = at_m$ *where* $a \in K_{m-1}$ *and some* A *which is algebraic over* K_{m-1} *such that* $L = K_m(A, T)$.

Proof. Similar to the proof of 2.2 in [5].

By induction on m and 2.2, we get

2.3 **Proposition.** $(K_m^c)_p$ *is generated by adjoining roots of the equations* $x^n = t_j$ *where* n *is any positive integer relatively prime to* p *and* $j = 1, \cdots, m$.

2.4 **Theorem.** $G(K_m) \cong_p \bigoplus_{j=1}^m \hat{Z}$.

Proof. We use $(K_j^c)_p$ instead of K_j^c for $j = 1, \cdots, m$ in the proof of 2.4 in [5]. Then the same proof works.

3. **The Brauer group of the Amitsur field** K_m. Denote the Brauer group of the field L by $B(L)$. The following theorem of Witt [6] is well known: Let L be a local field (i.e. a field which is complete in the topology induced by a discrete, nonarchimedean valuation) with residue class field $\bar{L}$. If $\bar{L}$ is perfect, then the sequence $\{1\} \to B(\bar{L}) \to B(\bar{L}) \to \chi(G(\bar{L})) \to \{1\}$ is exact and split. In case $\bar{L}$ is not perfect, the above result is no longer true. But if L and $\bar{L}$ have the same characteristic, then there is a similar theorem in [9] which is stated as follow: The sequence

$$\{1\} \to B(\bar{L}[t]) \to B(L) \to \chi(G(\bar{L})) \to \{1\}$$

is exact and split, where $B(\bar{L}[t])$ is the Brauer group of the polynomial ring in one indeterminant t over L. The relation between $B(\bar{L})$ and $B(\bar{L}[t])$ has been determined in [4, §7] and may be stated as: The sequence

$$\{1\} \to B'(\bar{L}[t]) \to B(\bar{L}[t]) \to B(\bar{L}) \to \{1\}$$

is exact and split, where $B'(\bar{L}[t])$ is a subgroup of $B(\bar{L}[t])(p)$.

Hence we have $B(\bar{L}[t]) \cong_p B(\bar{L})$. Now we have the following proposition analogous to the theorem of Witt.

3.1 **Proposition.** *Let* L *be a local field with* char $L =$ char $\bar{L}$ *and* $\bar{L}$ *is not perfect. Then the sequence*:

$$\{1\} \to (B(L))_p \to (B(L))_p \to (\chi(G(\bar{L})))_p \to \{1\}$$

is exact and split.

In what follows we use induction on m and 3.1 to determine the structure of $B(K_m)_p$.

3.2 **Theorem.** *$B(K_m) \cong_p \bigoplus_{i=1}^{(m-1)m/2} Q/Z$ for all nonnegative integers m.*

Proof. We use 3.1 instead of the theorem of Witt and $\cong_p$ instead of $\cong$. Then the same proof in 3.1 of [5] works.

Notations are as in [5, §4].

3.3 **Theorem.** *$B(K_m)_p$ is generated by $\{(t_i^r, t_j)_n \mid 1 \le i < j \le m,\ 1 \le r < n,$ n is a positive integer which is relatively prime to $p\}$.*

Proof. [5, Lemmas 4.1, 4.2] are still true when n is relatively prime to p. Similarly the proof of Theorem 4.3 in [5] can be carried over here.

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DEPARTMENT OF MATHEMATICS AND STATISTICS, UNIVERSITY OF MASSACHUSETTS, AMHERST, MASSACHUSETTS 01002